Multi-hadron production at HERA

Sergei Chekanov (ANL)

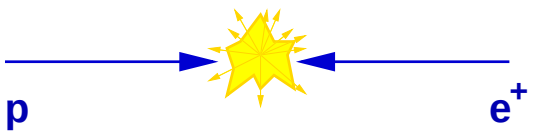
DESY Forum, Feb. 6 2001

- Introduction to particle spectra;
- pQCD in conjunction with LPHD hypothesis
- Part I: Single-particle spectra
- Part II: Many-particle spectra
- Conclusions

Description of hadronic final state

Inclusive distributions

$\rho(p_1, p_2, \ldots p_n)$



- probability density for n particles to be produced with $p_1, p_2, \ldots p_n$ irrespective of the presence of any other particles

- single-particle density $\rho(p_1)$

.. has been measured in many variables ($dn/d\eta$, dn/dp_t, dn/x_p etc)

$$\int_\Omega \rho(p_1)dp_1 = \langle n \rangle = \sum_{n=1}^{\infty} nP_n(\Omega) \quad -\textbf{mean multiplicity}$$

- two-particle density $\rho(p_1, p_2)$

$$\int_\Omega \int_\Omega \rho(p_1, p_2)dp_1 dp_2 = \langle n(n-1) \rangle = \sum_{n=2}^{\infty} n(n-1)P_n(\Omega)$$

$-\textbf{2nd} \quad \textbf{factorial} \quad \textbf{moment}$

- many-particle densities - by analogy

Many-particle densities can be studied from:

- **Shape of P_n, factorial moments $\langle n(n-1)...(n-k+1) \rangle$, cumulants, bunching parameters etc.**
- **Correlation functions, say $R = \rho(p_1, p_2)/\rho(p_1)\rho(p_2)$**

Some remarks:

- Most of experimental observables are related to $\rho(p_1, p_2, \ldots p_n)$
  - require counting separate particles (usually charged)
  - collinear and infrared sensitive (depend on the QCD cut-off used in the perturbative calculations)
- this is in contrast to some observables which can be constructed such that they are collinear and infrared safe (jets, event shapes, energy-energy correlations)
- Are they important ?
  - Better understanding of QCD
  - The confinement problem
  - Search for new physics beyond the Standard Model relies on our knowledge of multiple gluon activity
- DIS provides a unique possibility to confront both the analytic results and MC models with data at **various energy scales**

General pQCD approach to $\rho(p_1, p_2, \ldots p_n)$

Use the DGLAP equation and adopt it to describe the parton shower

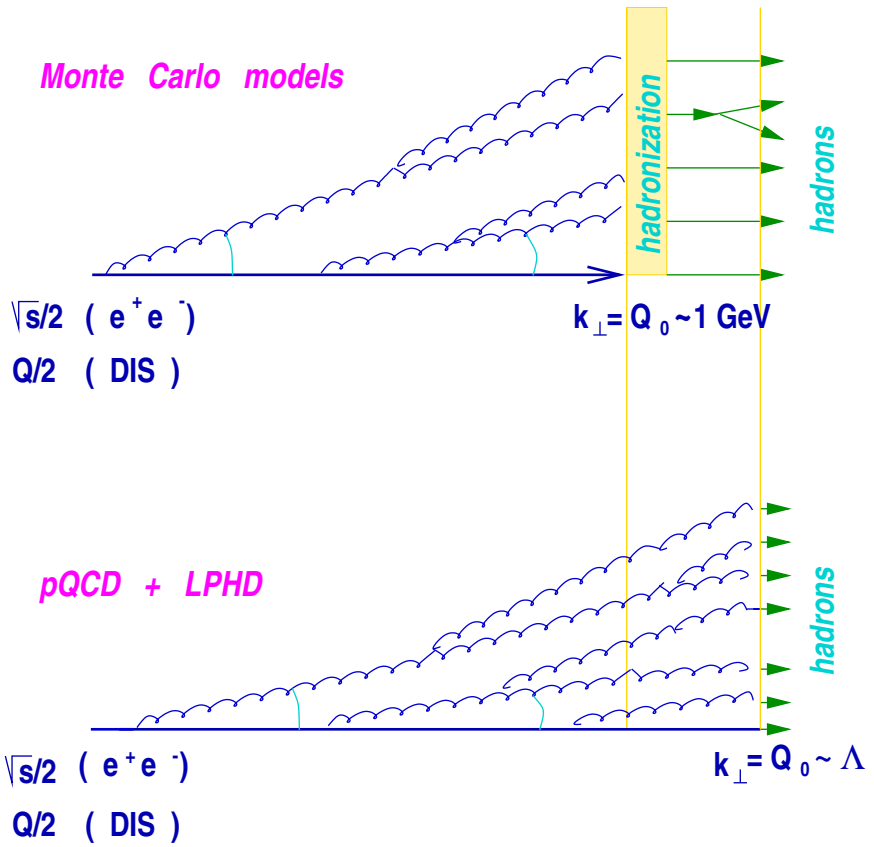
$\rightarrow$ can be regarded as the Markov process

$\rightarrow$ interference effects lead to *angular ordering*

- DLLA (Double Leading Log Approximation)
- MLLA (Modified Leading Log Approximation)

$\rightarrow$ apply the LPHD (Local Parton-Hadron Duality)

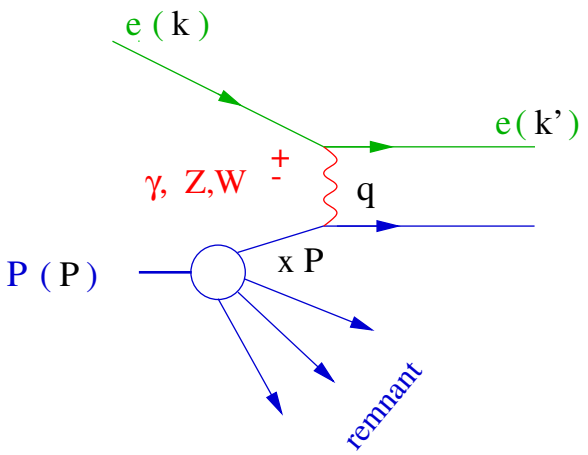
LPHD and QCD



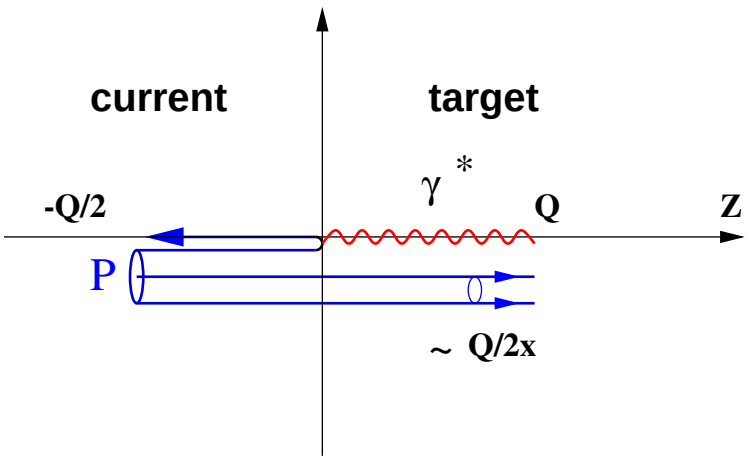
Analytic QCD and LPHD:

- Works for most inclusive observables at high energy;
- QCD cascade is evolved towards a scale $Q_0 \sim \Lambda$ (a few 100 MeV);
- Ignores hadronisation dynamics;
- MC cross-check: set $Q_0 \sim \Lambda$ and switching off hadronization.

Kinematics of NC DIS



Breit Frame



$Q^2 = -q^2 = -(k-k')^2$ squared 4-momentum transfer;

$x = Q^2/2p\cdot q$ fractional parton momentum $(0 \le x \le 1)$;

$y = p\cdot q/(p\cdot k)$ fractional energy transfer $(0 \le y \le 1)$;

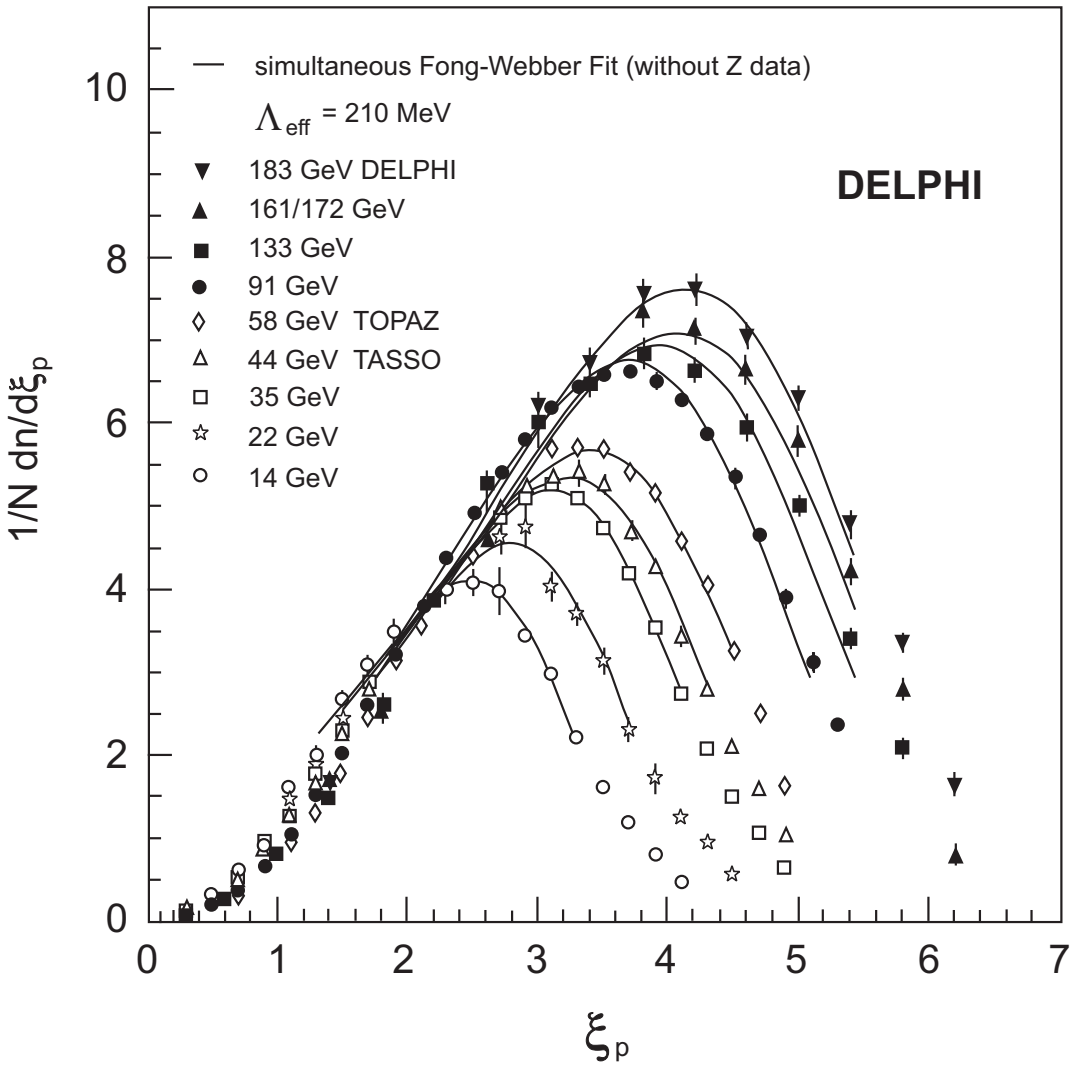
$W^2 = (q+p)^2 \simeq \frac{Q^2}{x}(1-x)$ $\gamma^* P$ Center-of-Mass

Single-Particle Spectra

ξ distribution

$$\xi_p = \ln \frac{1}{x_p}, \qquad x_p = \frac{p}{E_{jet}}$$

$$p \downarrow \Longrightarrow \xi_p \uparrow, \qquad \xi_p^{max} = \ln \frac{E_{jet}}{\Lambda}$$



Single-Particle Spectra

$x_p = 2p^{Breit}/Q$ distribution in the current region of the Breit frame

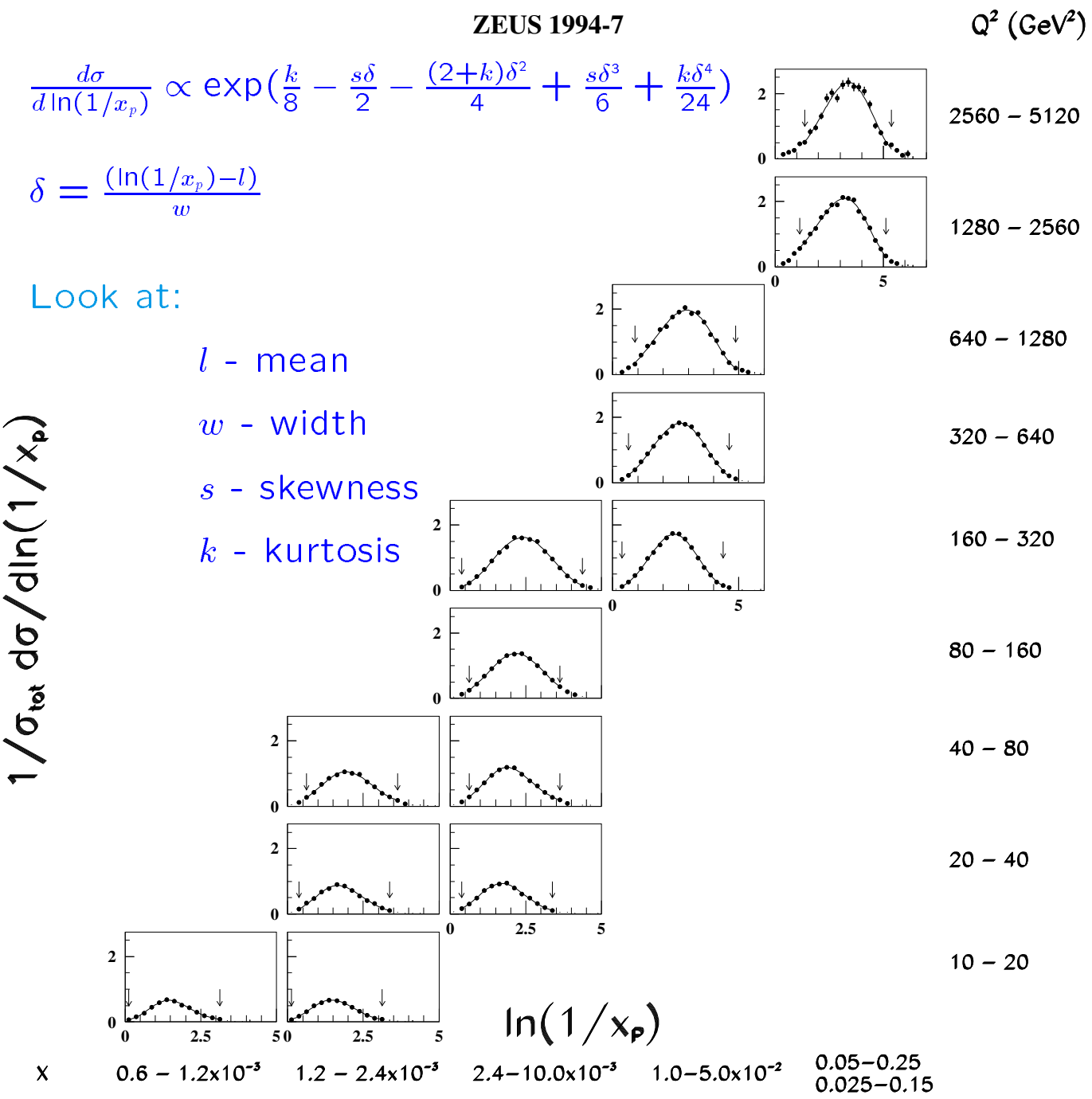
ZEUS Coll, Eur. Phys. J C11 (1999) 251

$$\frac{d\sigma}{d\ln(1/x_p)} \propto \exp\left(\frac{k}{8} - \frac{s\delta}{2} - \frac{(2+k)\delta^2}{4} + \frac{s\delta^3}{6} + \frac{k\delta^4}{24}\right)$$

$$\delta = \frac{(\ln(1/x_p) - l)}{w}$$

Look at:

- l - mean
- w - width
- s - skewness
- k - kurtosis



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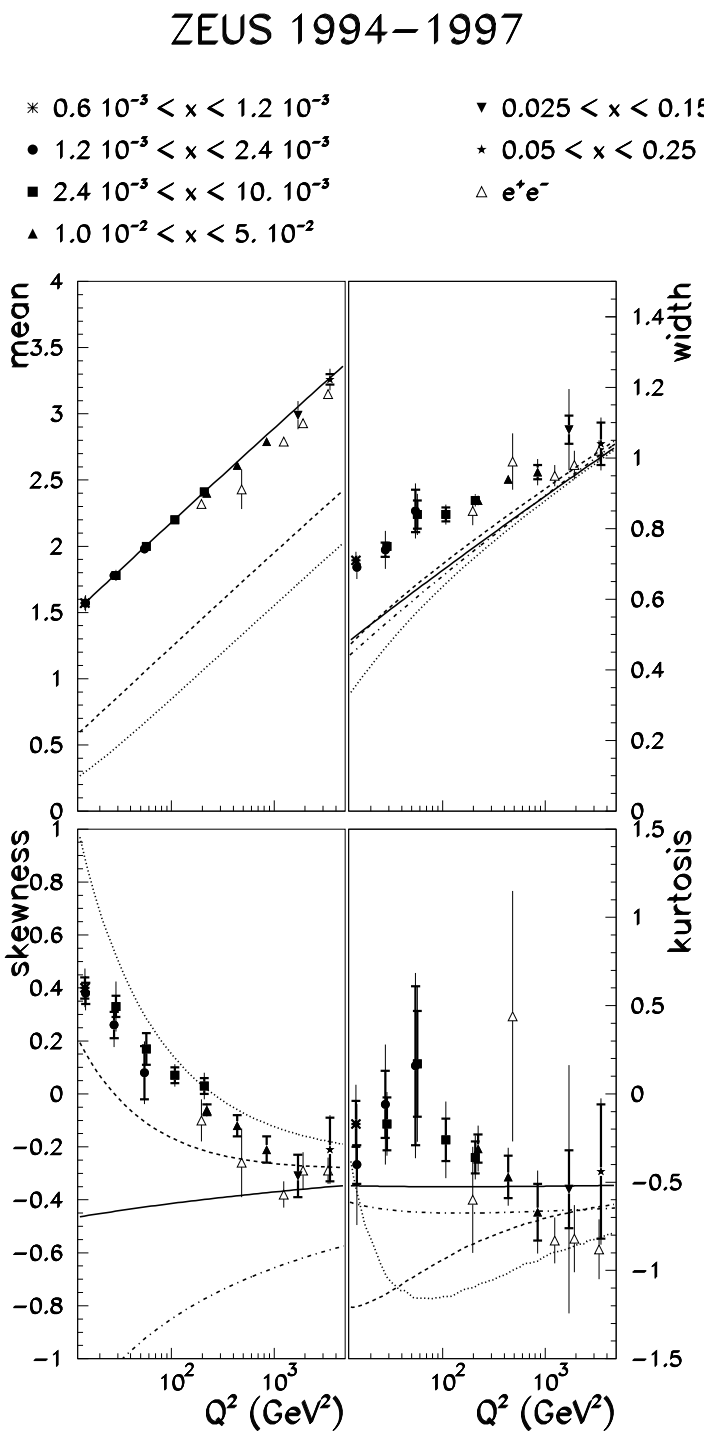

Agreement for $Q_0 = \Lambda$ at high Q^2

Single-Particle Spectra

$x_p = 2p^{Breit}/Q$ distribution in the current region of the Breit frame

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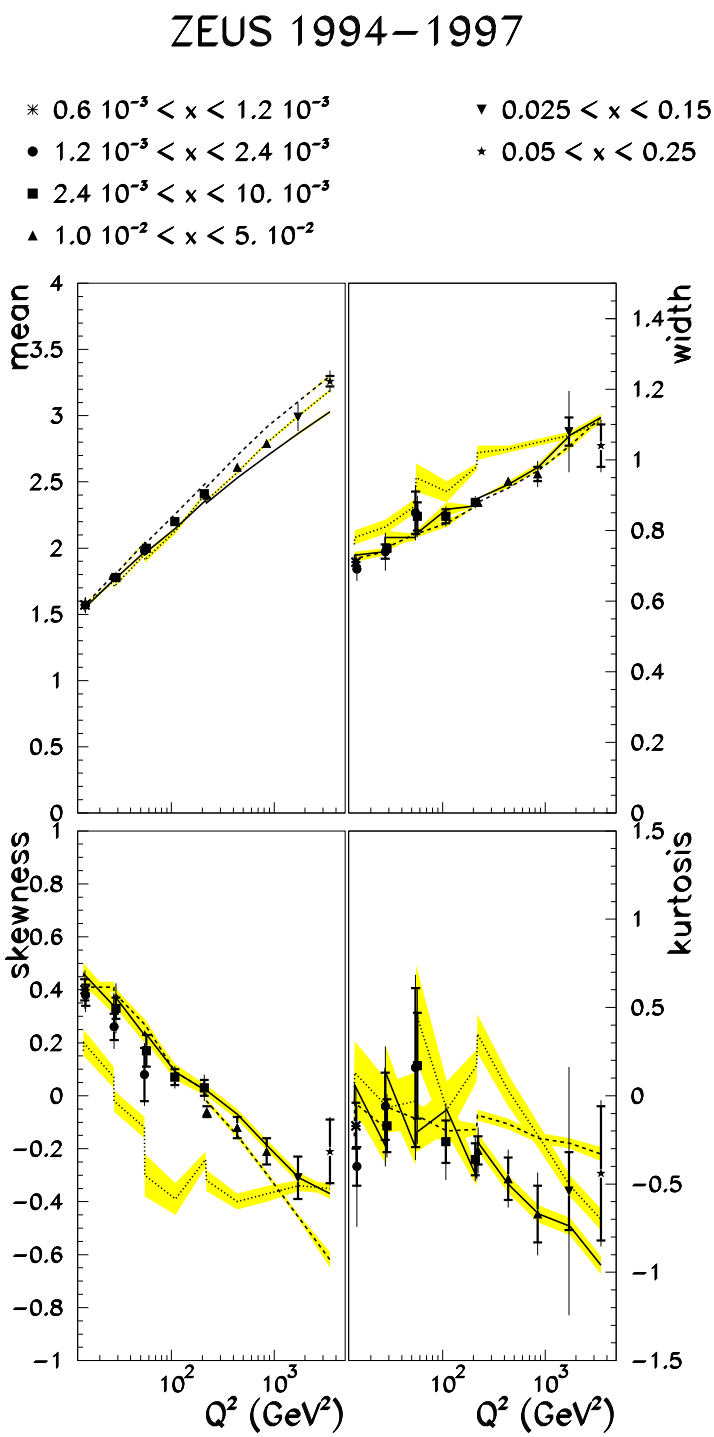


ARIADNE

LEPTO SCI

HERWIG

Reproduce energy dependence

Some discrepancies

Single-Particle Spectra

$x_p = 2p^{Breit}/Q$ distribution in the current region of the Breit frame

Before $\ln(p_h/P) \equiv \ln(1/x_p)$ ($P = Q/2$ for DIS)

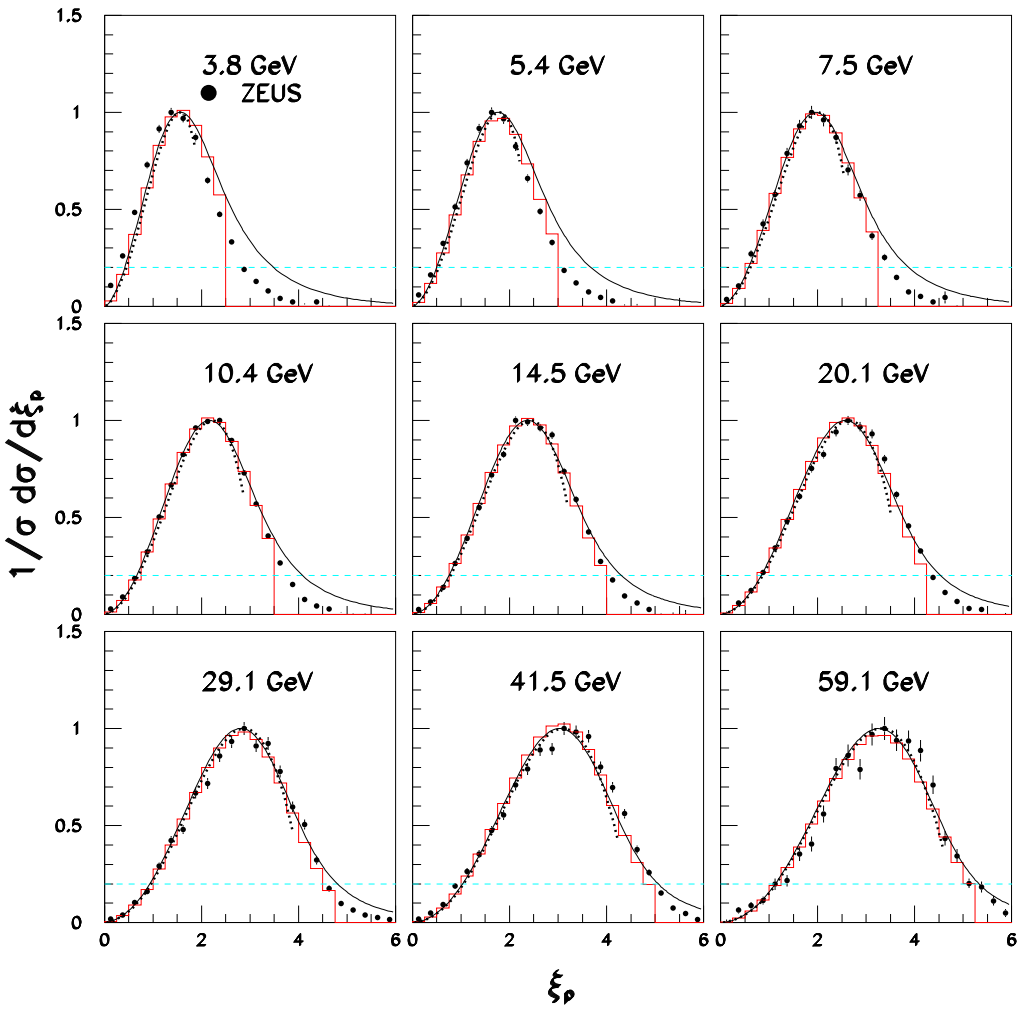
But predictions for $\ln(E_h/E) \equiv \ln(1/x_e)$ ($E = Q/2$ for DIS)

terminated at $E_{min} = Q_0/E$ (or Λ/E)

Need for kinematic correction ($E_h \neq p_h$):

V.Khoze,S.Lupia,W.Ochs, PL B386 (1996) 451

N.H.Brook, I.O.Skillicorn, hep-ph/0011218



$$\frac{1}{N}\frac{dn_h}{d\xi_p} \propto \frac{p_h}{E_h} D(\xi, Y)$$

$$\xi = \ln \frac{E}{\sqrt{E_0^2 e^{-2\xi_p} + m_{eff}^2}}$$

MLLA-M

MLLA-0

Fit: Λ=280 MeV
m_{eff}=230 MeV

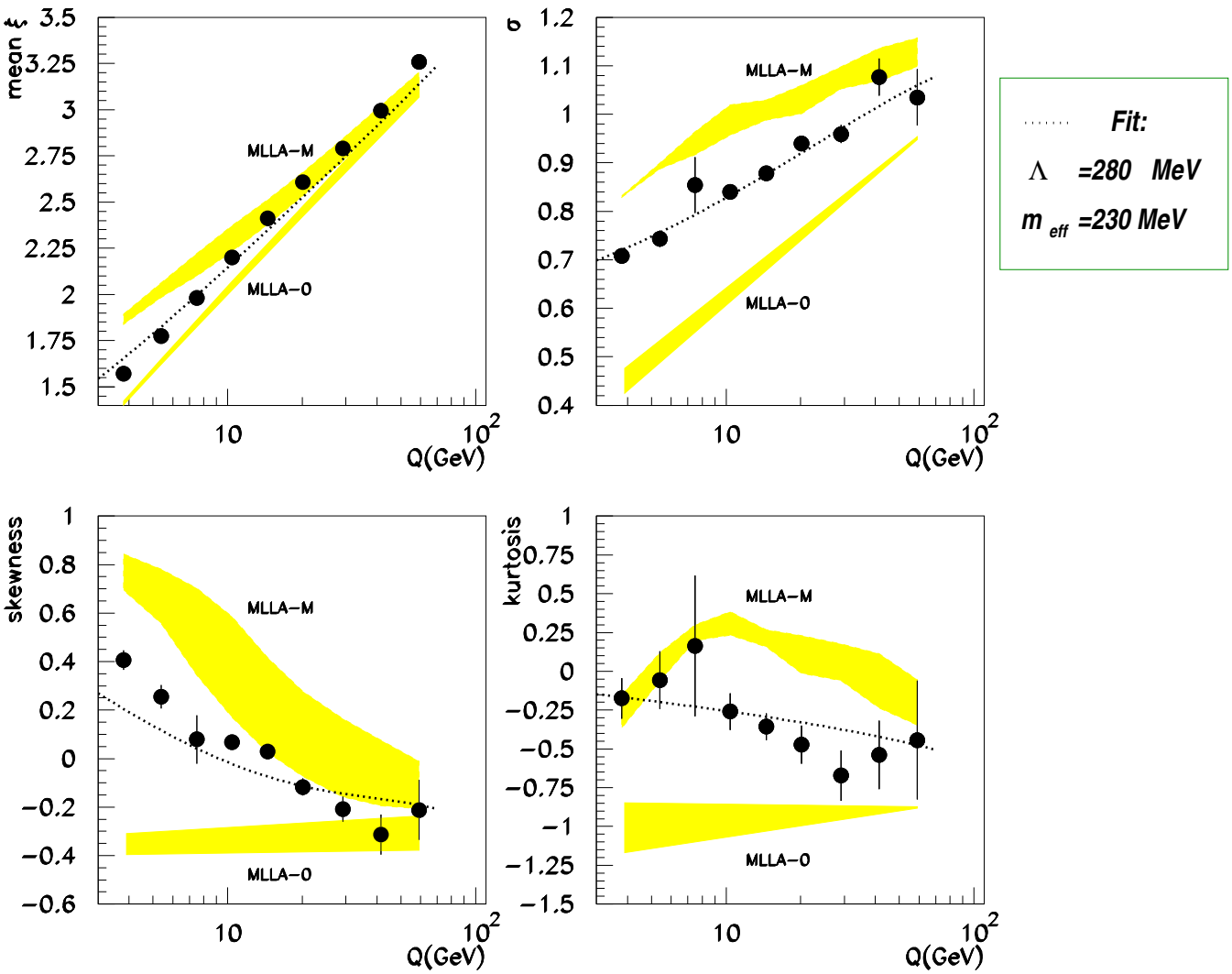
Single-Particle Spectra

$x_p = 2p^{Breit}/Q$ distribution in the current region of the Breit frame

N.H.Brook, I.O.Skillicorn, hep-ph/0011218

Yu.Dokshitzer, V.Khoze, S.Troyan:
J. Mod. Phys. A7 (1992) 1875
MLLA-0: $Q_0 = \Lambda = 280$ MeV

V.Khoze,S.Lupia, W.Ochs, PL B386 (1996) 451
MLLA-M: $Q_0 = \Lambda = m_{eff} = 280$ MeV



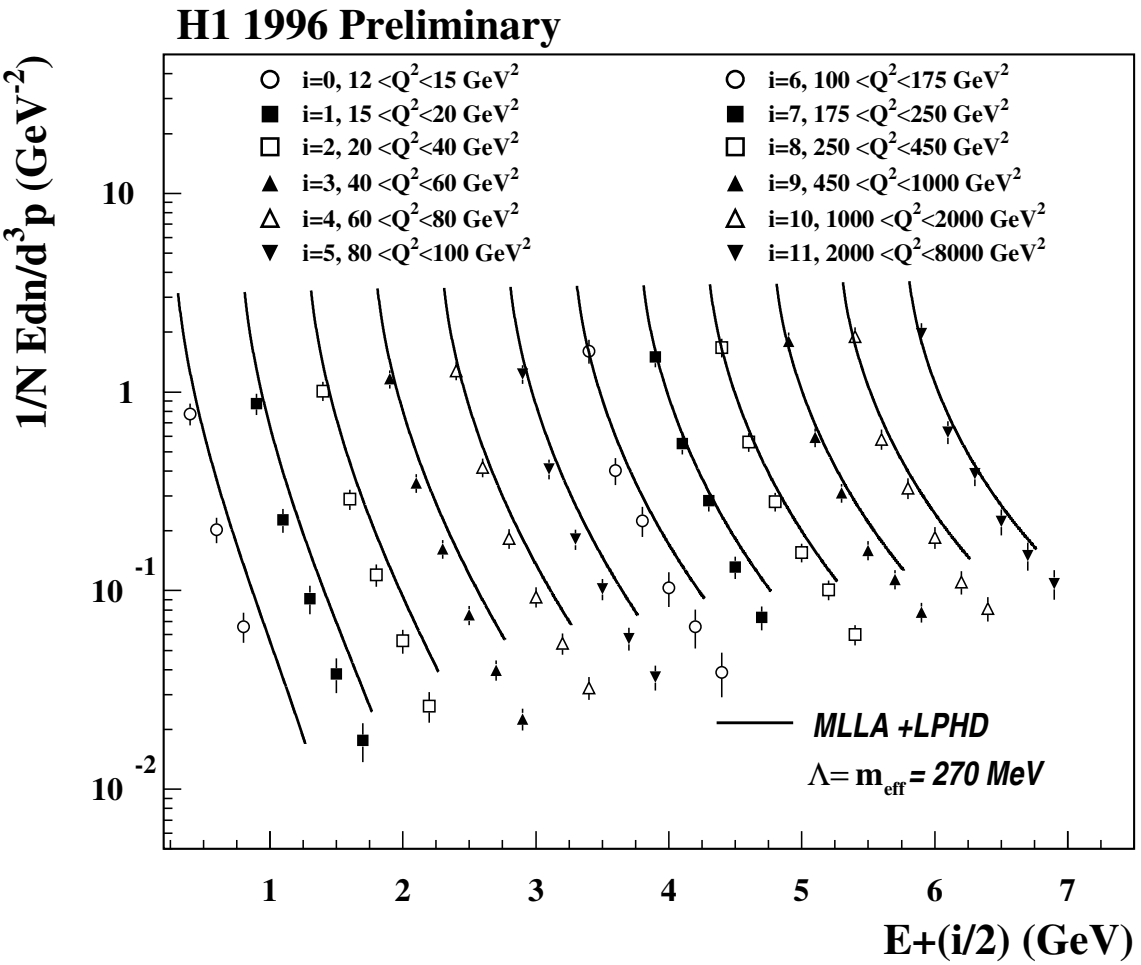
MLLA-0 and MLLM-M bracket the data

Single-Particle Spectra

Invariant Energy Spectra: V.Khoze, S.Lupia, W.Ochs, PL B394 (1997) 179

$$E\,\frac{dn}{d^3p} \propto \frac{3C_A}{\pi(11N_C - 2n_f)}\,\frac{E-Q_0}{\lambda\, Q_0\, p^2} \qquad \lambda = \ln\frac{Q_0}{\Lambda},$$

H1 Coll., paper 531, 29th Conf. on HEP, Vancouver, Canada (1998)



Good agreement with MLLA+LPHD at high Q^2

Current-Target Interdependence

are current and target regions interdependent ?

For e^+e^- two hemispheres are independent

Look at coefficient of correlations:

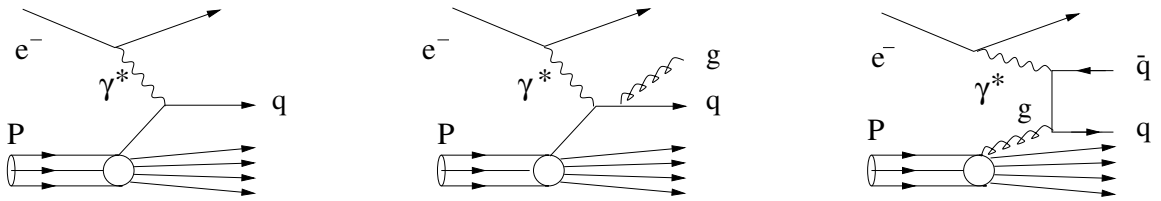
$$\kappa = s_c^{-1} s_t^{-1} \mathrm{cov}(n_c, n_t), \quad \mathrm{cov}(n_c, n_t) = \langle n_c n_t \rangle - \langle n_c \rangle \langle n_t \rangle$$

n_c - number of particles in the current region
s_c - standard deviation in the current region
n_t - number of particles in the target region
s_t - standard deviation in the target region

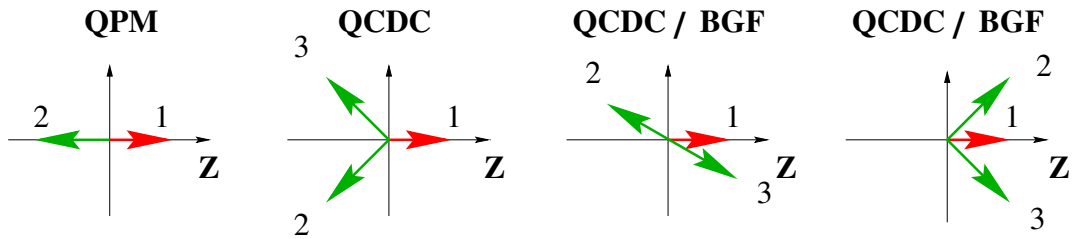
First-order QCD expectations (SC, J.Phys. G25 (1999) 59):

$$\mathrm{cov}(n_c, n_t) \sim -N_{BGF}/N_{tot}$$

- Large anti-correlations due to BGF processes
- Expected to be small at high Q^2
- DIS in the Breit frame differs from e^+e^-



a) QPM **b) QCDC** **c) BGF**



source for anticorrelations

Current-Target Correlations

coefficient of correlations between current and target regions

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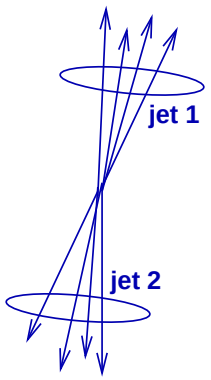
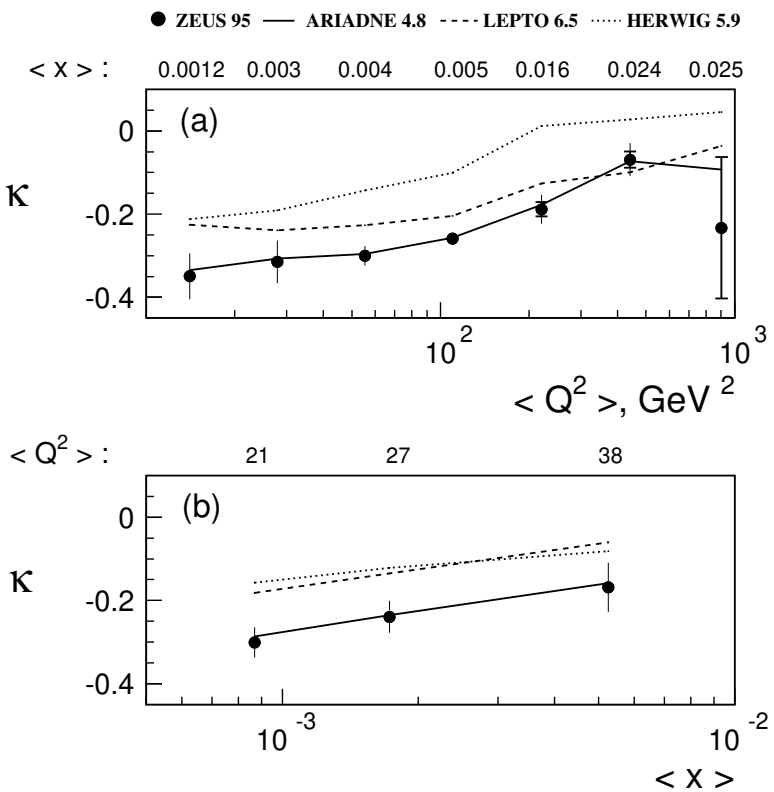


Coefficient of correlations

$\kappa \sim 0$ for e^+e^-



Particles in current and target regions correlate at low Q^2

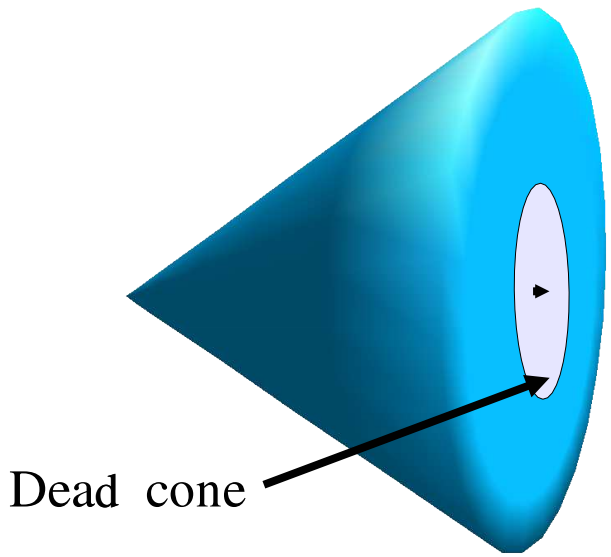
At large Q^2 the current region is similar to a single hemisphere of e^+e^-

Soft-gluon angular screening ("dead cone")

So far we assumed $m_q \sim 0$

In case of heavy quarks we should expect:

- Gluon emission is reduced within $\Theta_0 = m_q/E_q$
- Unlike to QED, never was observed in QCD ...

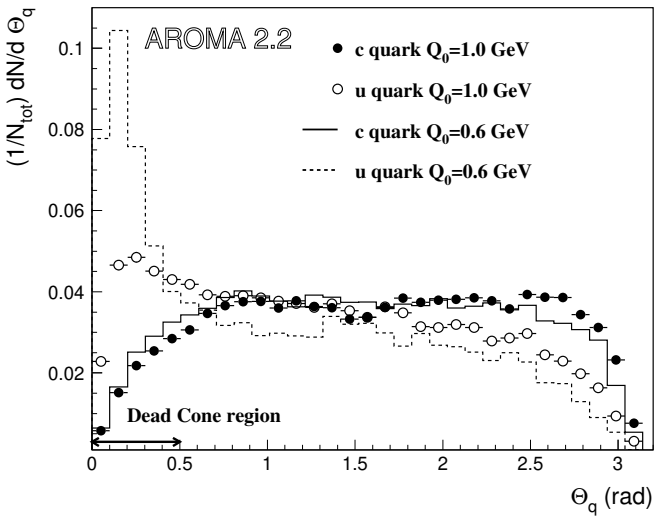


For charm quark: At LEP : $\Theta_0 = 2^o - 4^o$
At HERA: $\Theta_0 = 20^o - 30^o$

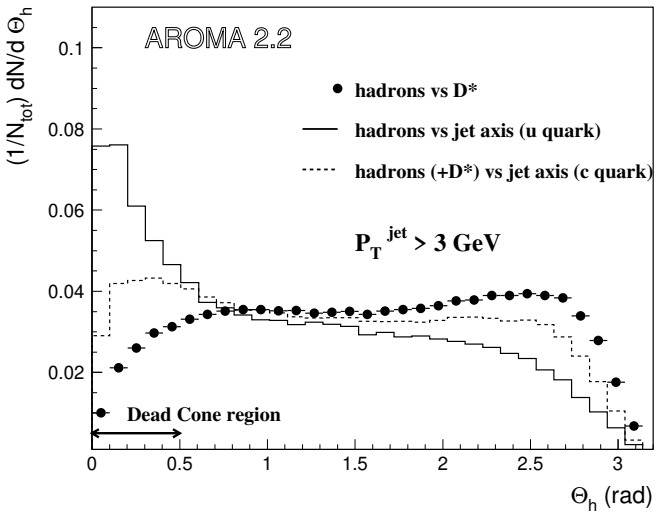
Soft-gluon angular screening ("dead cone")

S.C., Phys.Lett. B484 (2000) 51-57

Parton-level Monte Carlo expectations :



.. can be accessed through hadron-level observables:



Conclusions: Part 1

What did we learn from single-particle spectra ?:

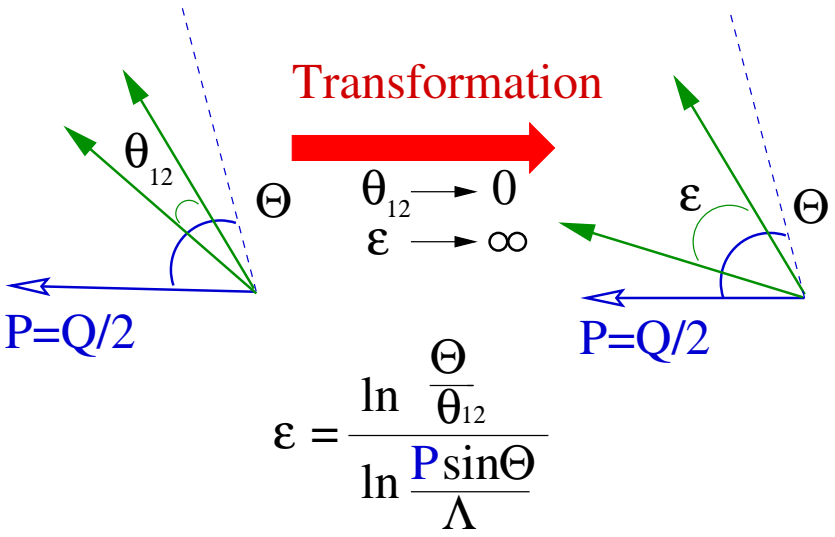
- MLLA+LPHD agrees well with HERA data for $Q^2 \gtrsim 400$ GeV^2

- MLLA+LPHD fails at lower energies:
  o kinematic corrections reduce discrepancies ($m_{eff} \neq 0$)
  o depend on approximations at low momentum regions

- Are we sure that low Q^2 DIS events are safe for comparison ?
  - At $Q^2 \gtrsim 400$ GeV^2 the current-target correlations are small $\rightarrow$ properties of e^+e^- events
  - At $Q^2 \lesssim 400$ GeV^2 the current-target correlations are large:
    $\rightarrow$ explains larger width of x_p spectra in the current region ?

- HERA has large potential to study soft-particle spectra:
  - Some cuts should be used to increase similarity of DIS event kinematics with e^+e^- at low Q^2
  - Dead cone effect never has been measured
    $\rightarrow$ HERA provides an unique opportunity to do this.

Two-Particle Density in the Current Region



$$\rho(\varepsilon) = \frac{1}{N_{\rm ev}}\frac{dn_{\rm pair}}{d\varepsilon} \qquad \hat{Y} \equiv -\frac{\ln[\rho(\varepsilon)/(b\langle n(\Theta)\rangle^2)]}{2\sqrt{\ln[P\sin\Theta/\Lambda]}}$$

pQCD in DLLA + LPHD :

$$\hat{Y} \simeq 2\beta(1-0.5\omega(\varepsilon))$$

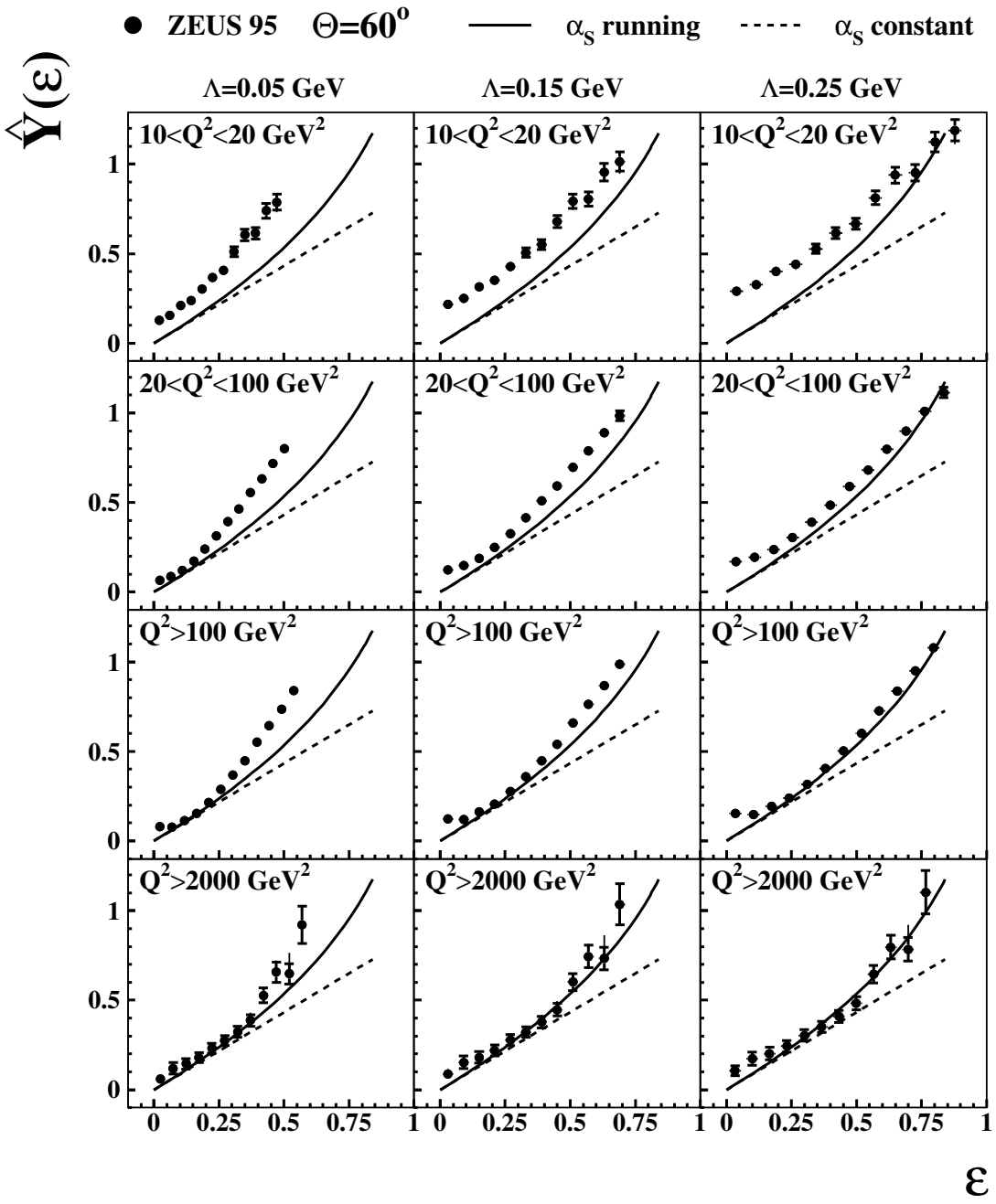
$$\omega(\varepsilon) = 2\sqrt{1-\varepsilon}\,(1-\ln(1-\varepsilon)/8)$$

$$\beta = 6(11\,N_c - 2n_f)^{-1/2}, \qquad b = 2\beta\sqrt{\ln(P\sin\Theta/\Lambda)}$$

W.Ochs, J.Wosiek, PL B289 (1992) 159,B304 (1993) 144

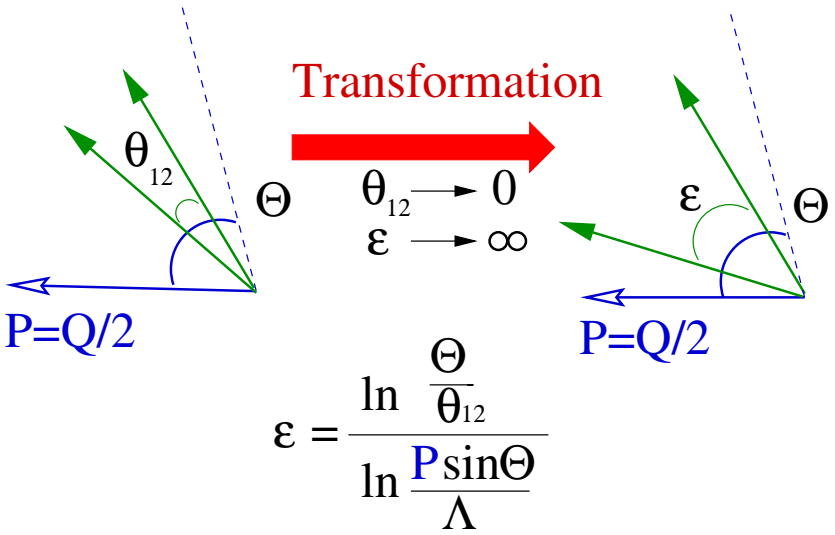
Two-Particle Density in the Current Region

ZEUS Coll., EPJ C12 (2000) 53



Good description at high Q^2 and $\Lambda > 0.15$ GeV

Two-Particle Correlations in the Current Region



$$\rho(\varepsilon) = \frac{1}{N_{\rm ev}} \frac{dn_{\rm pair}}{d\varepsilon} \qquad Y \equiv \frac{\ln[\rho(\varepsilon)/\rho_{\rm mix}(\varepsilon)]}{\sqrt{\ln(P\sin\Theta/\Lambda)}}$$

pQCD in DLLA + LPHD :

$$Y \simeq 2\beta(\omega(\varepsilon) - 2\sqrt{1-\varepsilon})$$

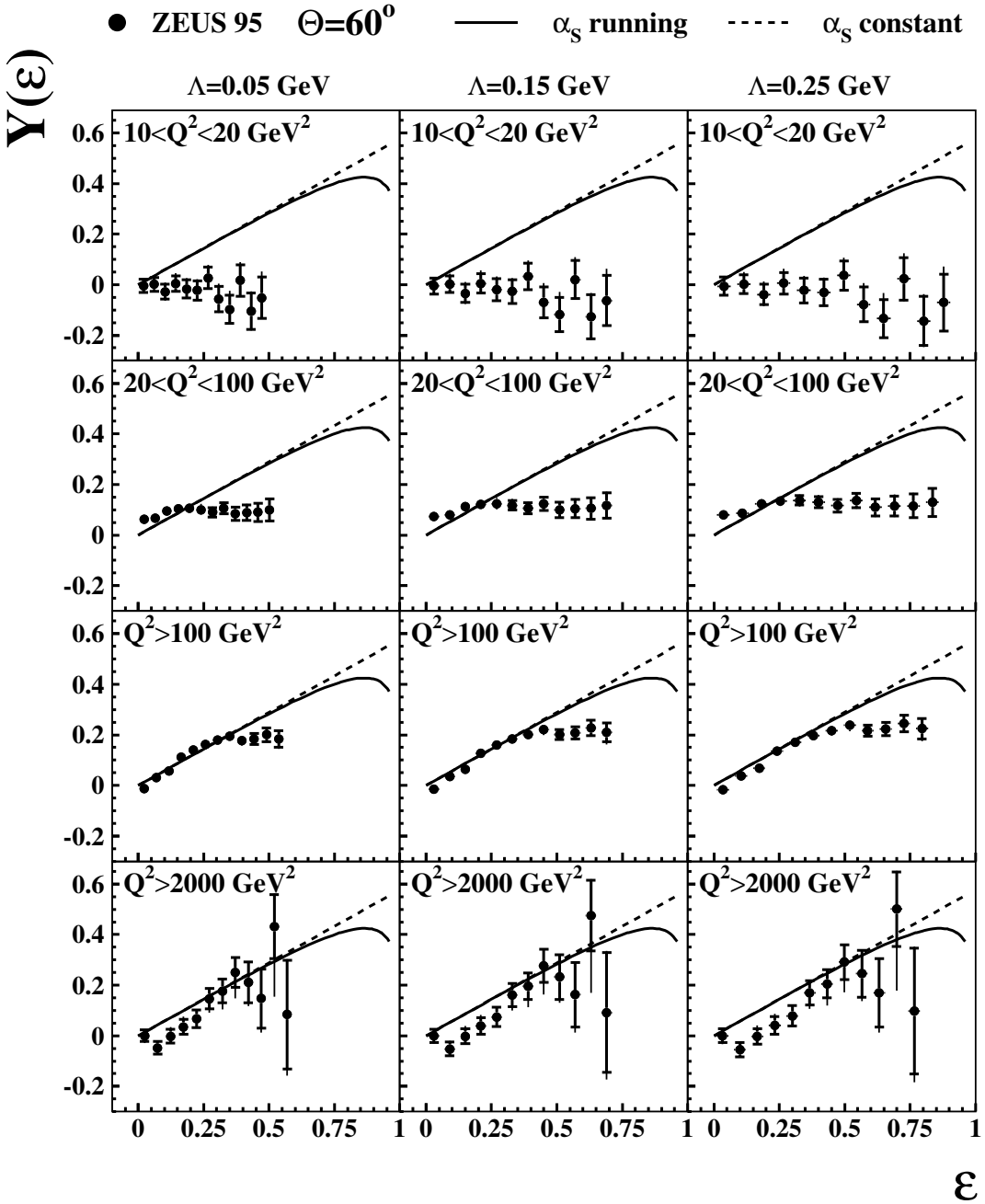
$$\omega(\varepsilon) = 2\sqrt{1-\varepsilon}\,(1 - \ln(1-\varepsilon)/8)$$

$$\beta = 6(11\,N_c - 2n_f)^{-1/2}, \qquad b = 2\beta\sqrt{\ln(P\sin\Theta/\Lambda)}$$

W.Ochs, J.Wosiek, PL B289 (1992) 159,B304 (1993) 144

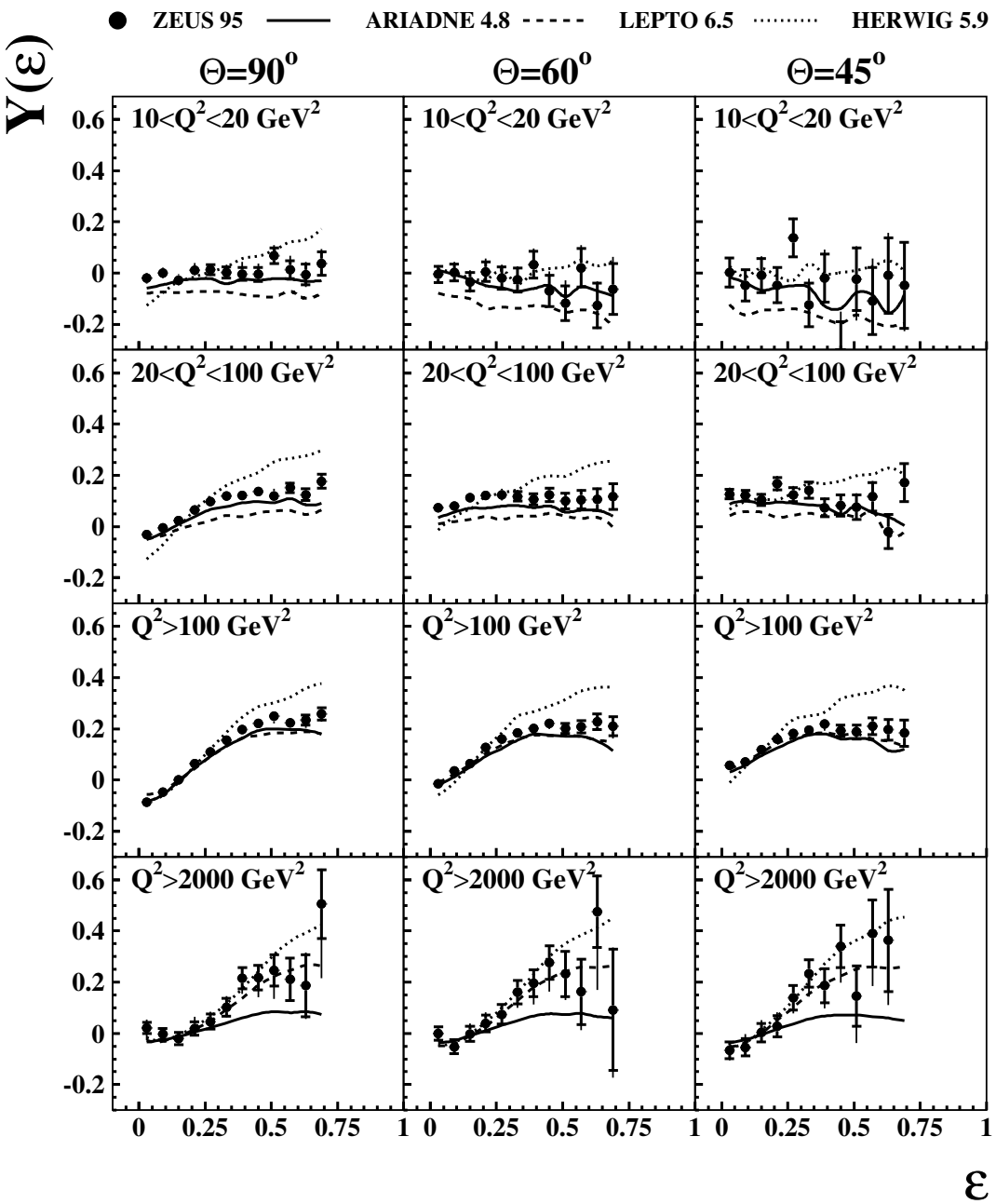
Two-Particle Correlations in the Current Region

ZEUS Coll., EPJ C12 (2000) 53



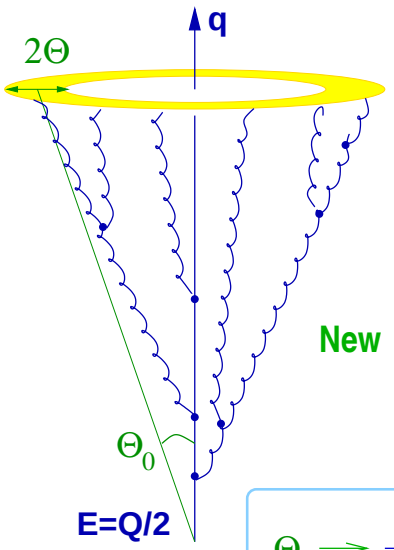
No satisfactory description by DLLA at low Q^2

Two-Particle Correlations in the Current Region (data vs MC)



All MC models show discrepancies

Angular Moments



$P_n(2\Theta)$
probability to find
"n" gluons in the ring
of size 2Θ at Θ_0

New variable: $z = \ln \frac{\Theta_0}{\Theta} \Big/ a$

$a = \ln \frac{E\Theta_0}{\Lambda} = \text{const}$

$\Theta \to \frac{\Lambda}{E}$ $\quad z \to 1$

$\Theta \to \Theta_0$ $\quad z \to 0$

Factorial moments:

$$F_q(z) = \langle n \rangle^{-q} \sum_{n=q}^{\infty} \frac{n!}{(n-q)!} P_n(z)$$

Experimental definition:

$$F_q(z) = \langle n \rangle^{-q} \langle n(n-1)\dots(n-q+1) \rangle$$

DLLA QCD :

$$\ln \frac{F_q(z)}{F_q(0)} = z\, a\, (1 - D_q)(q-1)$$

Fluctuations are determined by D_q
($D_q = 1$, $F_q(z) = F_q(0)$ - no phase-space correlations)

QCD calculations for D_q:

- fixed-coupling regime

$$D_q = \gamma_0(E\Theta_0)\frac{q+1}{q}, \qquad \gamma_0(E\Theta_0) = \sqrt{2\,C_A\alpha_s(Q)/\pi}$$

- running coupling $\alpha_s(Q)$ calculations:

Yu. Dokshitzer and I.M. Dremin, Nucl. Phys. B402 (1993) 139

$$(a): \quad D_q \simeq \gamma_0(Q)\frac{q+1}{q}\left(1+\frac{q^2+1}{4q^2}z\right)$$

Ph. Brax, J.-L. Meunier and R. Peschanski, Z. Phys. C62 (1994) 649:

$$(b): \quad D_q \simeq 2\,\gamma_0(Q)\frac{q+1}{q}\left(\frac{1-\sqrt{1-z}}{z}\right)$$

W. Ochs and J. Wosiek, Phys. Lett. B289 (1992) 159, Z.Phys. C68 (1995) 269:

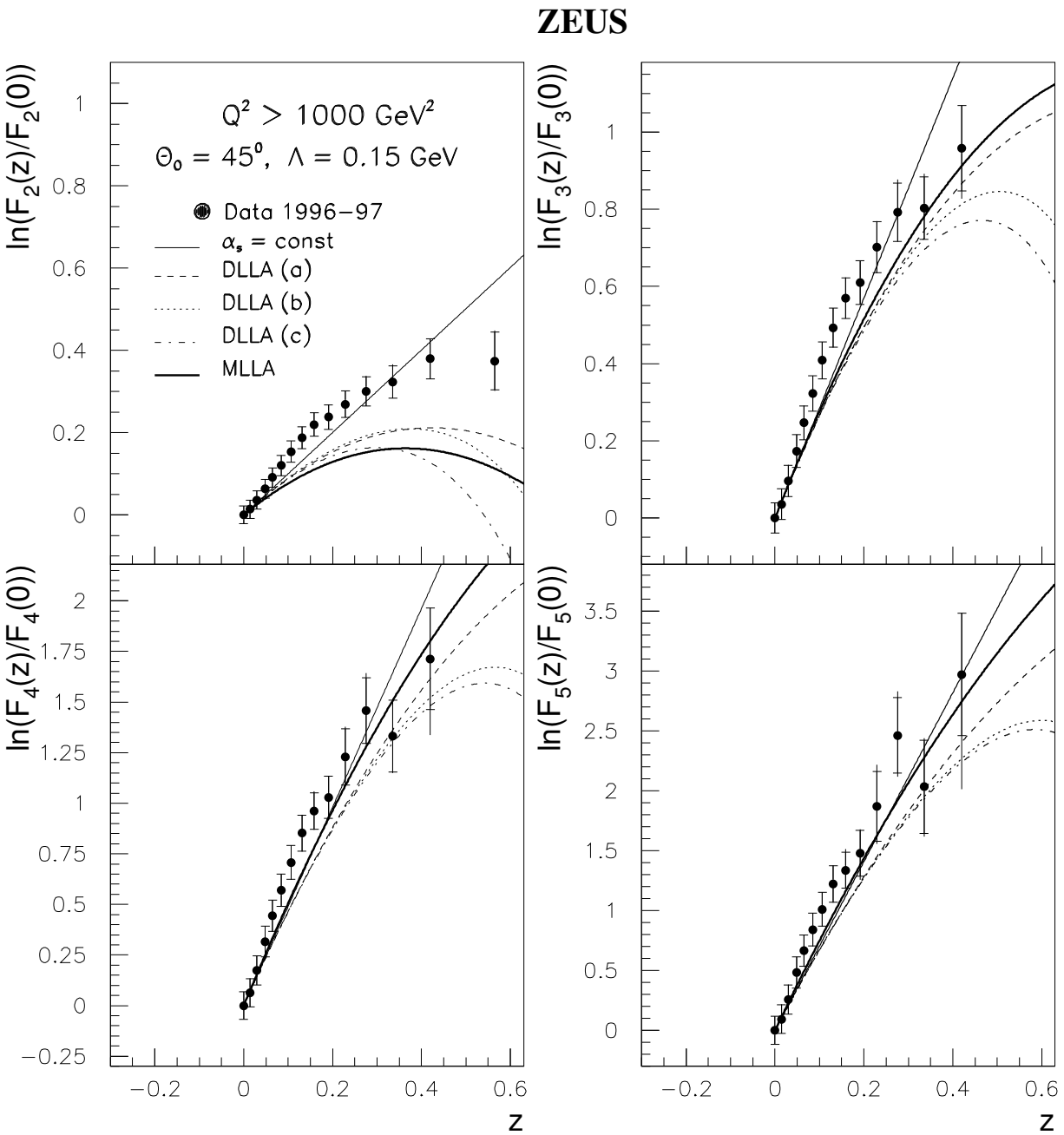
$$(c): \quad D_q \simeq 2\,\gamma_0(Q)\frac{q-w(q,z)}{z(q-1)}$$

$$w(q,z) = q\sqrt{1-z}\left(1-\frac{\ln(1-z)}{2q^2}\right).$$

MLLA corrections given in Nucl. Phys. B402 (1993) 139

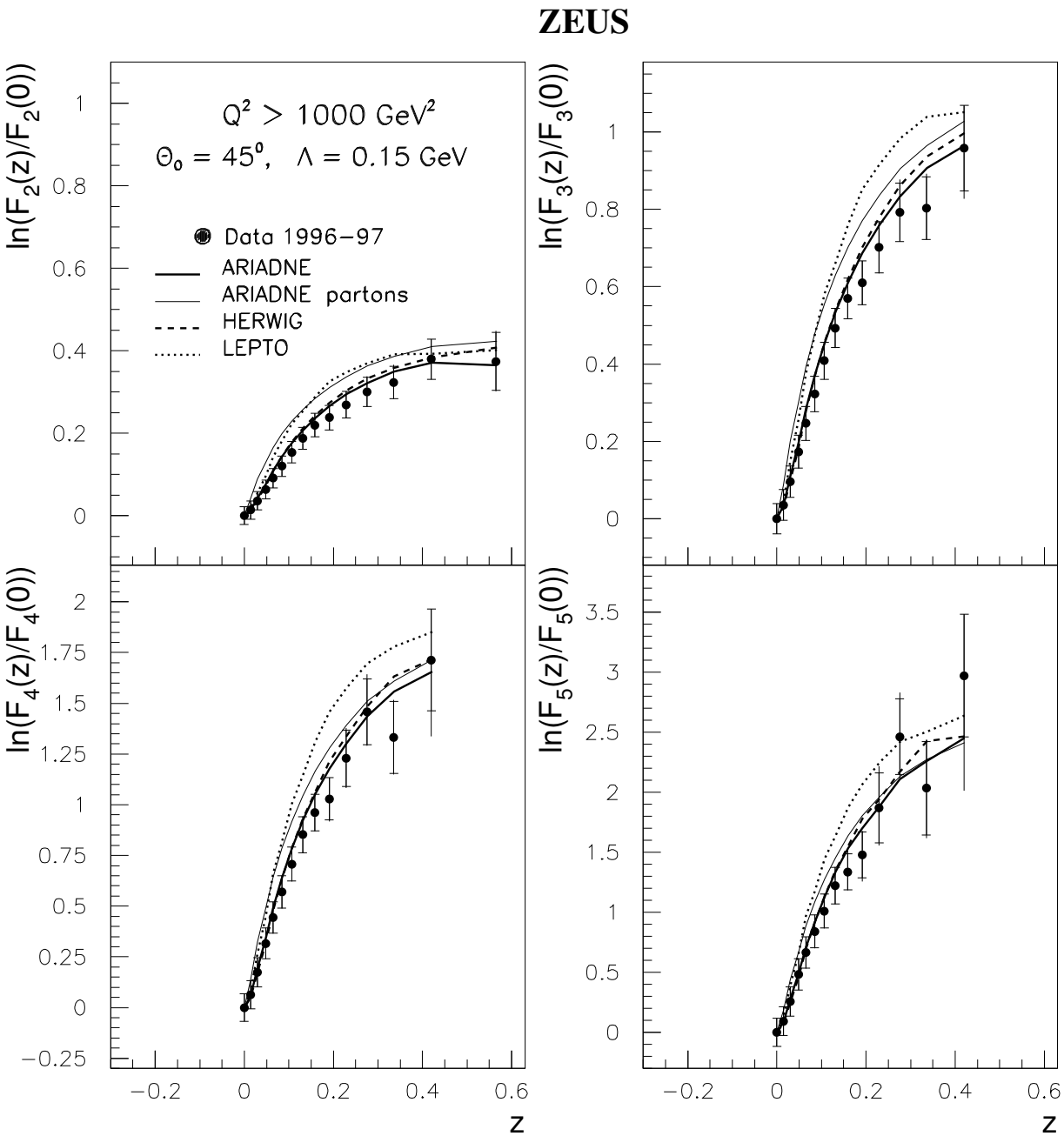
Angular Moments

ZEUS Coll., IHEP2000, Osaka, Abst 893, Plen 3, Par. 7

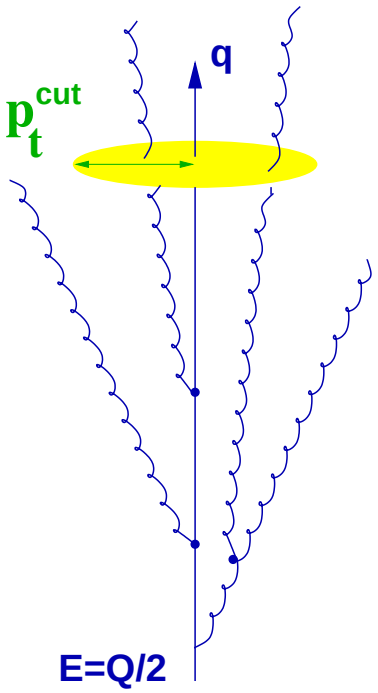


Angular Moments vs MC

ZEUS Coll., IHEP2000, Osaka, Abst 893, Plen 3, Par. 7



p_t^{cut} Moments



$P_n(p_t < p_t^{cut})$
probability to find
"n" partons with
$p_t < p_t^{cut}$

Factorial moments:

$$F_q(p_t < p_t^{cut}) = \langle n \rangle^{-q} \sum_{n=q}^{\infty} \frac{n!}{(n-q)!} P_n(p_t^{cut})$$

Experimental definition:

$$F_q(p_t < p_t^{cut}) = \langle n \rangle^{-q} \langle n(n-1)\dots(n-q+1) \rangle$$

n number of gluons with $p_t < p_t^{cut}$

DLLA QCD calculations :

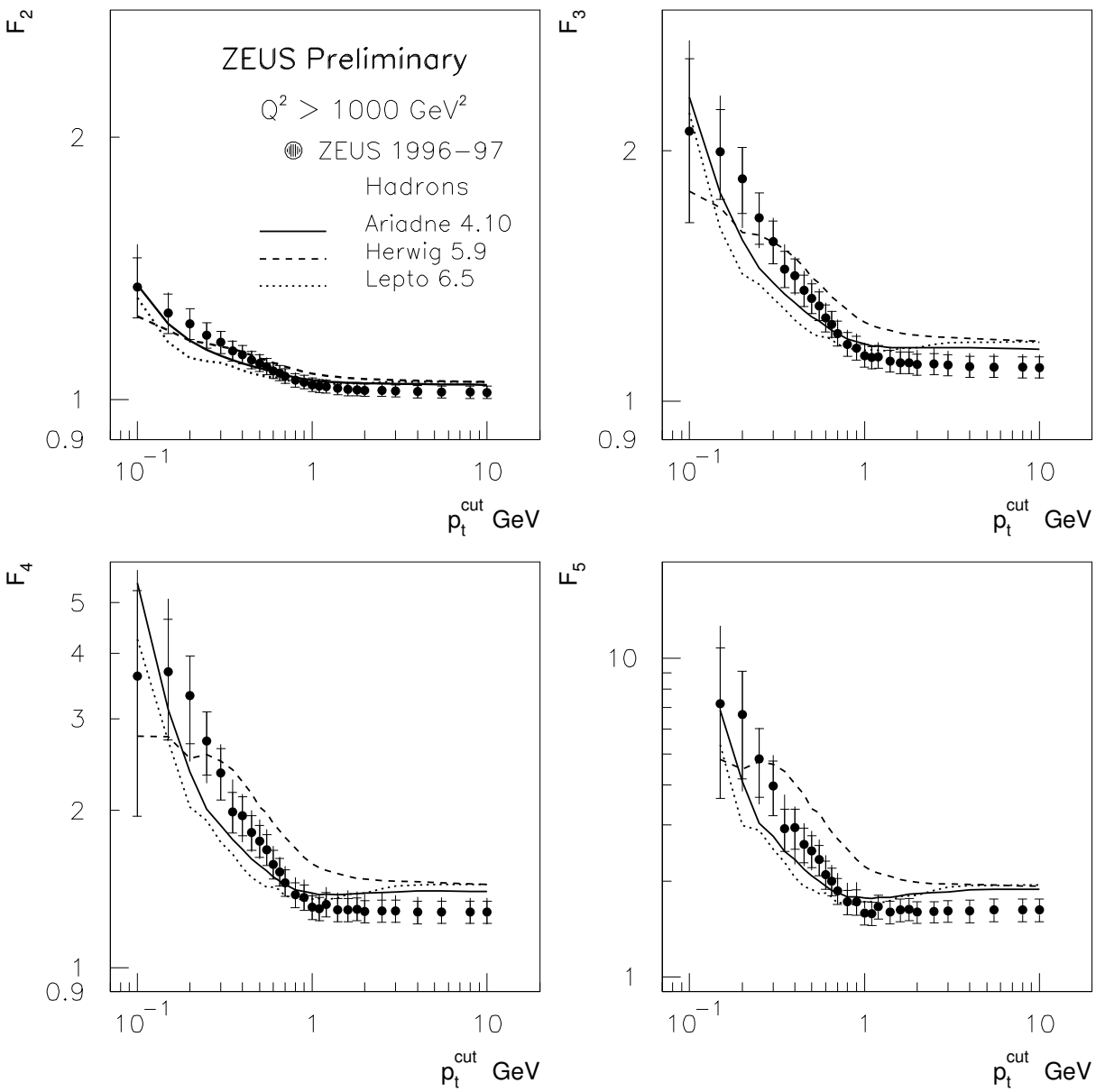
$$F_q(p_t^{cut}) \simeq 1 + \frac{q(q-1)}{6} \frac{\ln(p_t^{cut}/Q_0)}{\ln(E/Q_0)}$$

$F_q(p_t^{cut} \to Q_0) \to 1$ - Poisson statistics

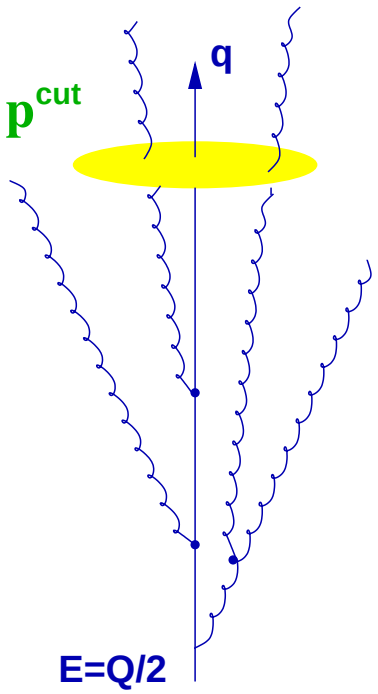
S.Lupia, W.Ochs, J.Wosiek, Nucl. Phys. B540 (1999) 405

p_t^{cut} Moments

Prelim, ZEUS Coll., IHEP2000, Osaka, Abst 893, Plen 3, Par. 7



p^{cut} Moments



$P_n(|\vec{p}| < p^{cut})$
probability to find
"n" partons with
$|\vec{p}| < p^{cut}$

Factorial moments:

$$F_q(p < p^{cut}) = \langle n \rangle^{-q} \sum_{n=q}^{\infty} \frac{n!}{(n-q)!} P_n(p^{cut})$$

Experimental definition:

$$F_q(p < p^{cut}) = \langle n \rangle^{-q} \langle n(n-1)\ldots(n-q+1) \rangle$$

n number of gluons with $p < p^{cut}$

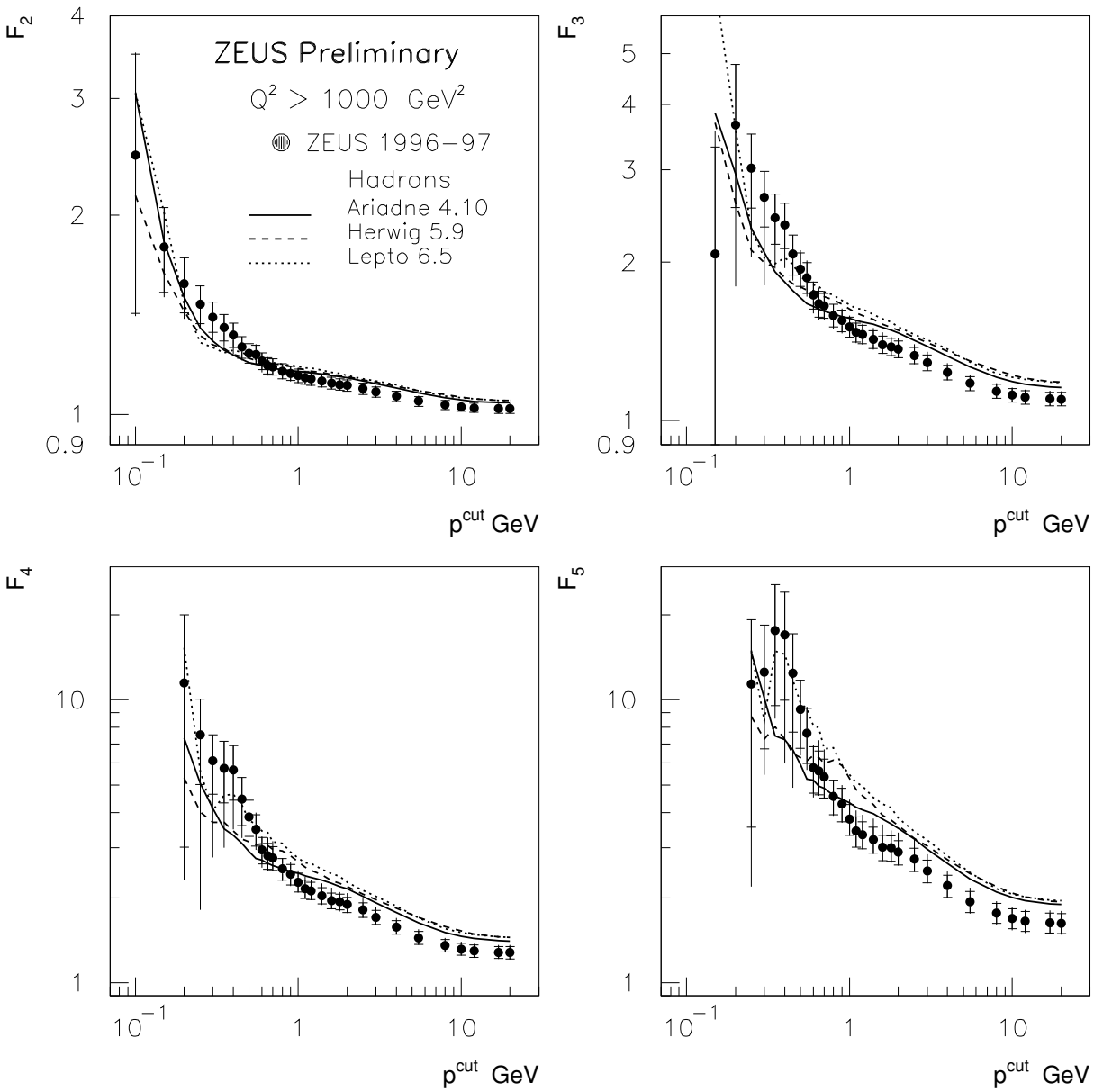
DLLA QCD :

$$F_q(p^{cut}) \simeq const$$

S.Lupia, W.Ochs, J.Wosiek, Nucl. Phys. B540 (1999) 405

p^{cut} Moments

Prelim, ZEUS Coll., IHEP2000, Osaka, Abst 893, Plen 3, Par. 7



Conclusions: Part 2

- **DLLA QCD + LPHD:**
  Reasonably agrees with 2-particle density at high Q^2;
  Shows some discrepancies with 2-particle correlations;
  o MLLA is needed

- **Multiplicity moments disagree with DLLA:**
  o for angular moments - quantitative differences;
  o p_t moments show opposite trends !

- **Is this related to approximations in DLLA ?**
  o possibly for angular moments (see MC parton level)
  o *unlikely* for p_t moments
  $\rightarrow$ parton-level ($\sim$ MLLA) shows the same discrepancies

- **Hadronisation has a large impact on correlations**
  o LPHD does not work for many-particle densities
  o many-particle inclusive densities are not sufficiently inclusive ?