Correlations and Fluctuations at LEP and HERA

Sergei Chekanov (ANL)

Durham, December 2001

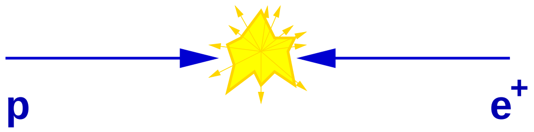
- Introduction to particle spectra
- pQCD in conjunction with LPHD hypothesis
- Universality between e^+e^- and DIS ?
- Genuine correlations and BE effect
- Correlations and analytic QCD:
  - Part I: Angular correlations
  - Part II: NFM in angular intervals
  - Part III: NFM restricted in p_T and p
- Conclusions

Description of hadronic final state

Inclusive distributions

$\rho(p_1, p_2, \ldots p_n)$

- probability density for n particles to be produced with $p_1, p_2, \ldots p_n$ irrespective of the presence of any other particles



• single-particle density $\rho(p_1)$

.. has been measured in many variables ($dn/d\eta$, dn/dp_t, dn/x_p etc)

$$\int_{\Omega} \rho(p_1) dp_1 = \langle n \rangle = \sum_{n=1}^{\infty} n P_n(\Omega) \quad \textbf{–mean multiplicity}$$

• two-particle density $\rho(p_1, p_2)$

$$\int_{\Omega}\int_{\Omega} \rho(p_1, p_2) dp_1 dp_2 = \langle n(n-1) \rangle = \sum_{n=2}^{\infty} n(n-1) P_n(\Omega)$$

$$\textbf{–2nd factorial moment}$$

• many-particle densities - by analogy

Many-particle densities can be studied from:

- **Shape of P_n, factorial moments $\langle n(n-1)...(n-k+1) \rangle$, cumulants, bunching parameters etc.**
- **Correlation functions, say $R = \rho(p_1, p_2)/\rho(p_1)\rho(p_2)$**

Some remarks:

- Many experimental observables are related to $\rho(p_1, p_2, \ldots p_n)$
  - require counting separate particles (usually charged)
  - collinear and infrared sensitive
- this is in contrast to some observables which can be constructed such that they are collinear and infrared safe (jets, event shapes, energy-energy correlations)
- Importance of correlations:
  - Better understanding of QCD / The confinement problem
  - Complementary to single-particle spectra and global event shapes
  - Search for new physics beyond the Standard Model relies on our knowledge on multiple gluon activity

General pQCD approach to $\rho(p_1, p_2, \ldots p_n)$

Adopt the DGLAP approach to describe the parton shower:

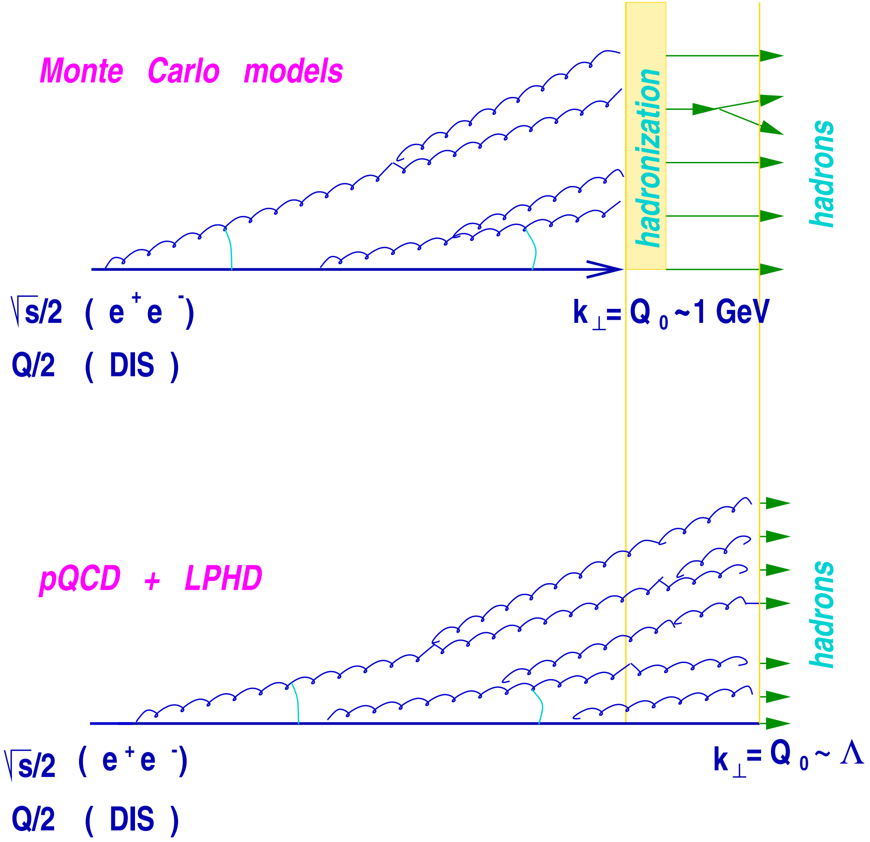
$\longrightarrow$ can be regarded as the Markov process

$\longrightarrow$ interference effects lead to *angular ordering*

- DLLA (Double Leading Log Approximation)
- MLLA (Modified Leading Log Approximation)

$\longrightarrow$ apply the LPHD (Local Parton-Hadron Duality)

LPHD and QCD



Analytic QCD and LPHD:

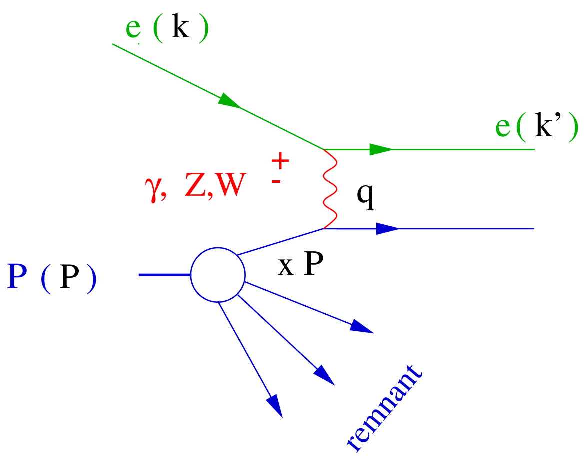
- Works for most inclusive observables at high energy;
- QCD cascade is evolved towards a scale $Q_0 \sim \Lambda$ (a few 100 MeV);
- Ignores hadronisation dynamics;
- MC cross-check: set $Q_0 \sim \Lambda$ and after removing hadronization.

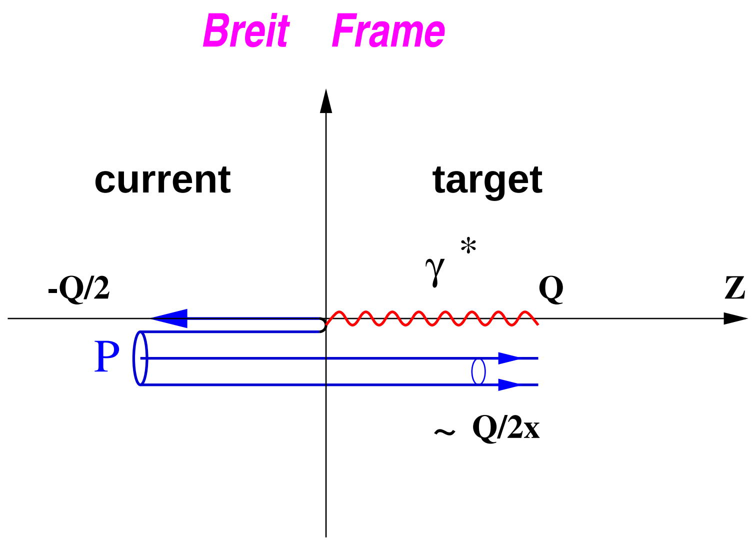
Experimental tests:
Find the direction of a quark (jet) and look at hadronic flow around it

Kinematics of e^+e^-

too simple to discuss ...

Kinematics of NC DIS





Q^2	$= -q^2 = -(k-k')^2$	squared 4-momentum transfer;
x	$= Q^2/2p \cdot q$	fractional parton momentum $(0 \leq x \leq 1)$;
y	$= p \cdot q/(p \cdot k)$	fractional energy transfer $(0 \leq y \leq 1)$;
W^2	$= (q+p)^2 \simeq \frac{Q^2}{x}(1-x)$	$\gamma^* P$ Center-of-Mass energy

Are quark hemispheres interdependent ?

where correlations are useful

coefficient of correlations

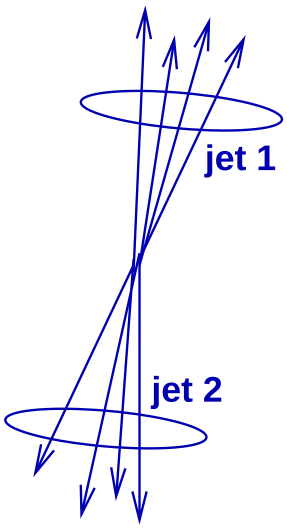
$$\kappa = s_1^{-1} s_2^{-1} \mathrm{cov}(n_1, n_2), \quad \mathrm{cov}(n_1, n_2) = \langle n_1 n_2 \rangle - \langle n_1 \rangle \langle n_2 \rangle$$

n_1 (n_2) - number of particles in first (second) hemisphere

s_1 (s_2) - standard deviation in first (second) hemisphere

or look at $\langle n_1 \rangle = a + b n_2$ (forward-backward (FB) correlations)

Two quark hemispheres are $\sim$ independent for e^+e^-:



Coefficient of correlations

$\kappa \sim 0$ for $e^+ e^-$

(TASSO and LEP results in terms of FB correlations)

Note: FB correlation is slightly positive due to $b\bar{b}$

Current-Target Correlations

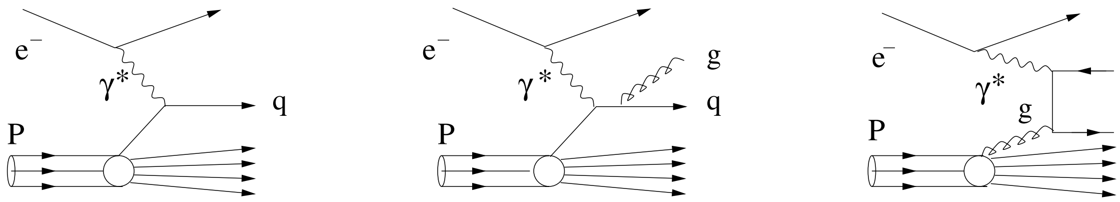
where correlations are useful

Look at correlations between current and target regions in DIS:

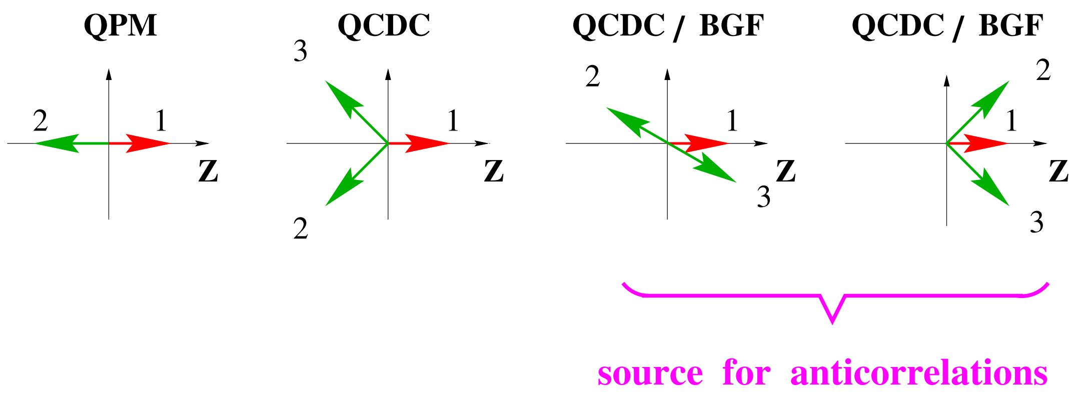
$$\kappa \propto -\sigma^{BGF} \sim -g(x)$$

(at LO QCD!)

- Anti-correlations due to boson-gluon fusion (BGF)
- Expected to be large at low Q^2

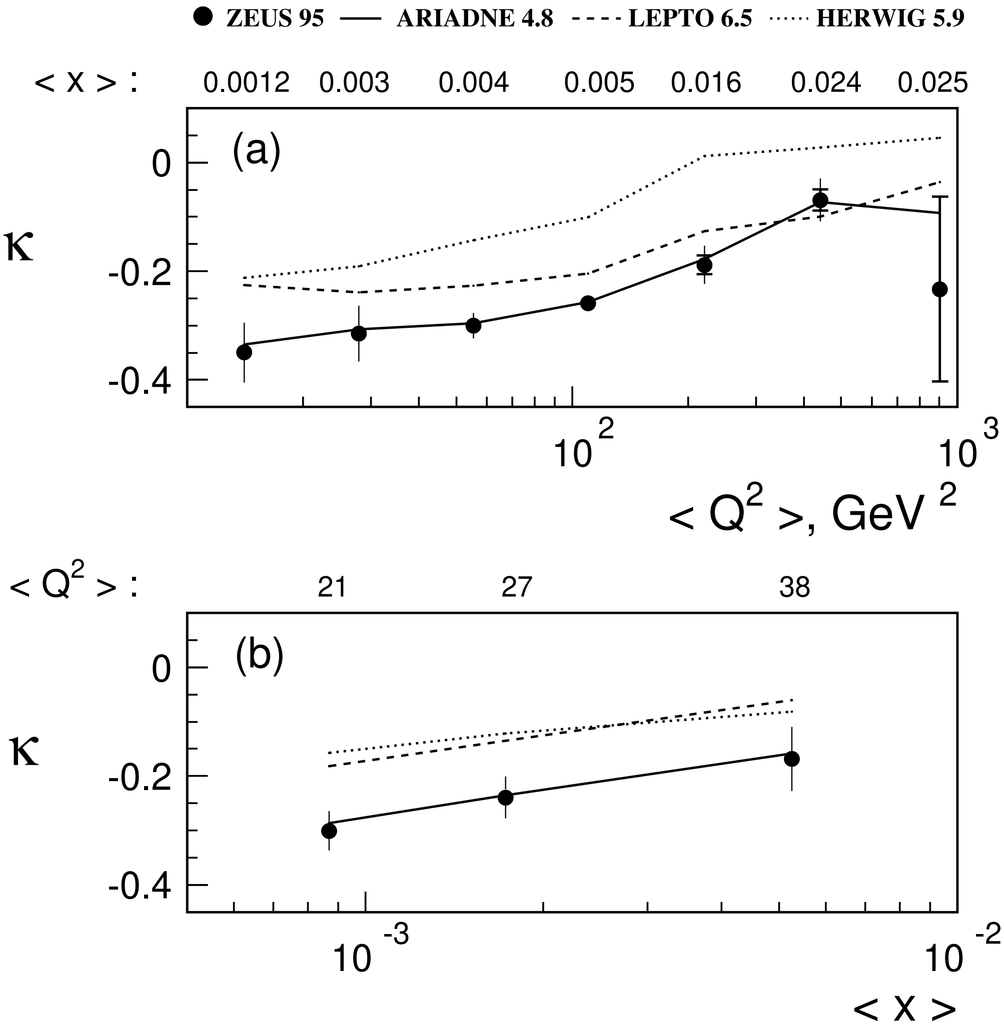


a) QPM　　b) QCDC　　c) BGF



Current-Target Correlations

ZEUS Coll, Eur. Phys. J C12 (2000) 55



Particles in current and target regions are correlated
Correlations are small at $Q^2 > 400$ GeV^2

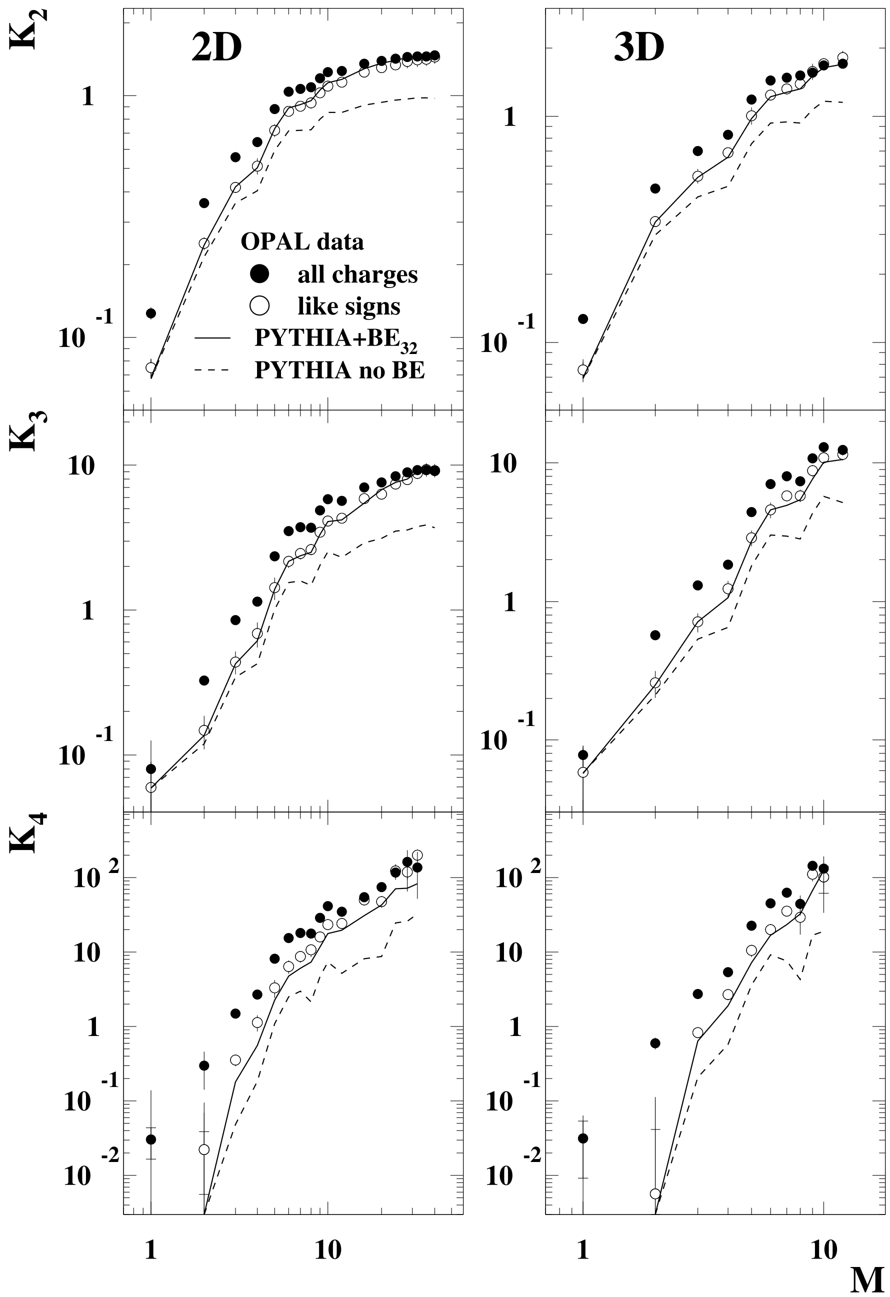
Genuine Correlations of Like-Sign Particles

where correlations are useful

Effects from final-state dynamics can be studied in great details:

Example: Bose-Einstein (BE) interference

Factorial cumulants (see OPAL Coll,CERN-EP-2001-070 for details)



large sensitivity to the BE correlations !

Some conclusions:

- Are low Q^2 DIS events safe for comparison with e^+e^- ?
  - at $Q^2 \gtrsim 400$ GeV^2 the current-target correlations are small :
    o property of e^+e^- events
    o small uncertainties from gluon density : good for QCD studies
  - large correlations at $Q^2 \lesssim 400$ GeV^2 $\rightarrow$ No universality

Nevertheless:

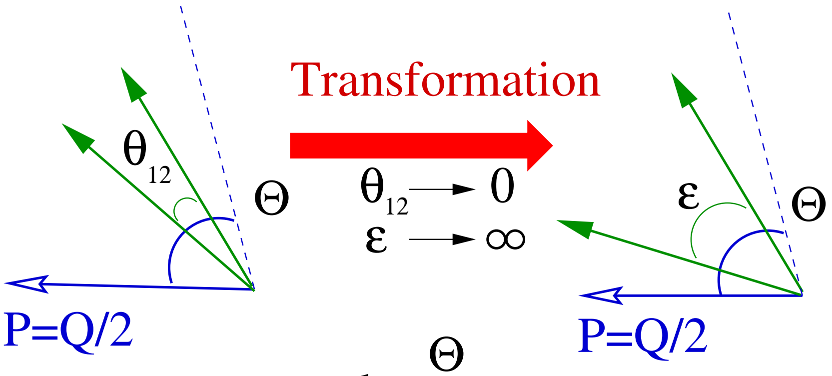
- DIS provides a unique possibility to confront both the analytic results and MC models with data at **various energy scales**, but
  - Comparisons between e^+e^- and DIS are safe for $Q^2 \gtrsim 400$ GeV^2
  - Some cuts should be used to increase similarity of the current region of DIS with a single hemisphere of e^+e^-

Now look at correlations within:

- a single hemisphere of e^+e^-
- current region of the Breit frame in DIS

Remark: for comparisons with the analytic QCD below, Bose-Einstein and forward-backward correlations are small (or removed)

Two-Particle Density



$$\varepsilon = \frac{\ln \dfrac{\Theta}{\theta_{12}}}{\ln \dfrac{P \sin\Theta}{\Lambda}}$$

$$\rho(\varepsilon) = \frac{1}{N_{\text{ev}}} \frac{dn_{\text{pair}}}{d\varepsilon} \qquad \hat{Y} \equiv -\frac{\ln[\rho(\varepsilon)/(b\langle n(\Theta)\rangle^2)]}{2\sqrt{\ln[P \sin\Theta/\Lambda]}}$$

pQCD in DLLA + LPHD :

$$\hat{Y} \simeq 2\beta(1 - 0.5\omega(\varepsilon))$$

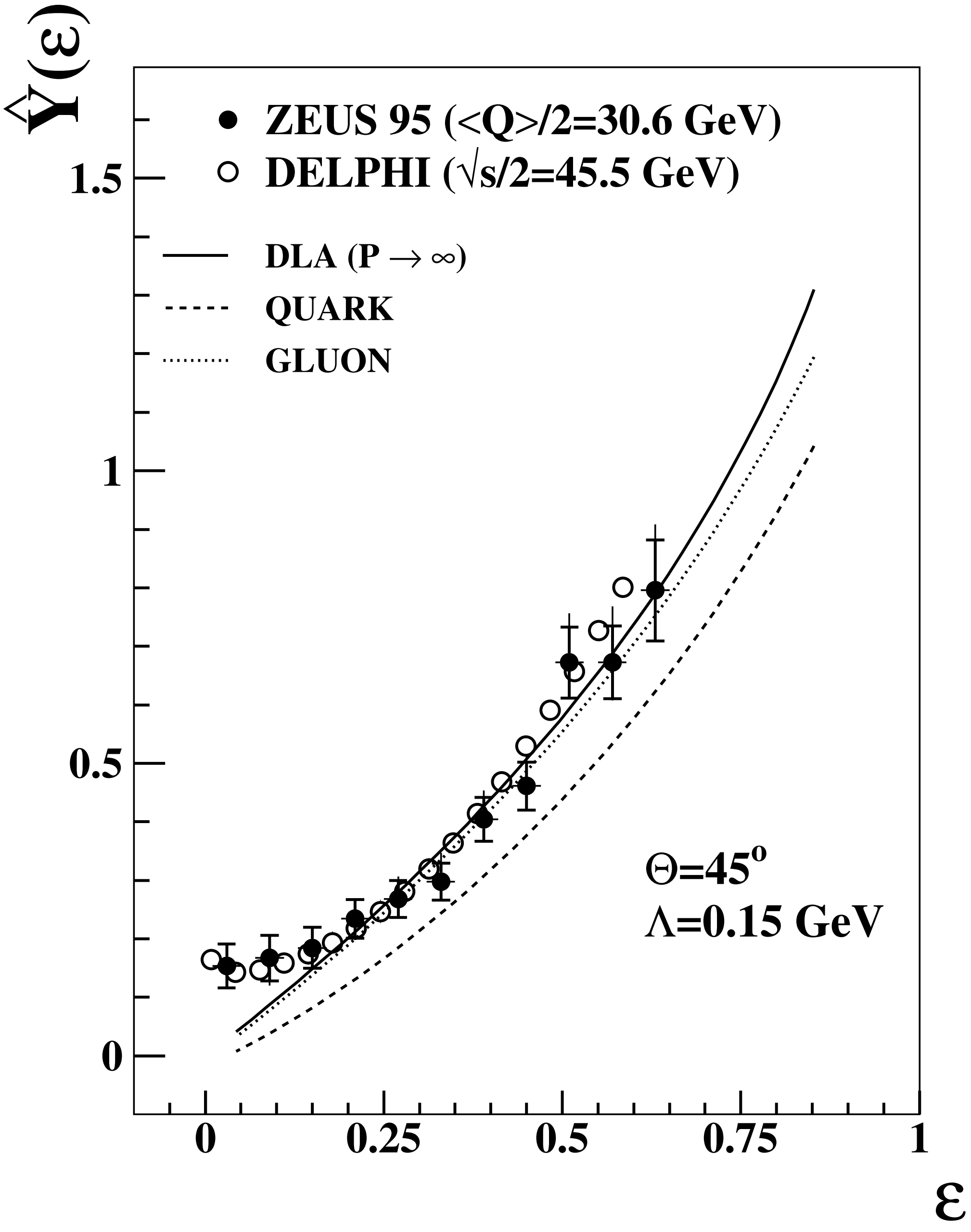
$$\omega(\varepsilon) = 2\sqrt{1-\varepsilon}\ (1 - \ln(1-\varepsilon)/8)$$

$$\beta = 6(11\,N_c - 2n_f)^{-1/2}, \qquad b = 2\beta\sqrt{\ln(P \sin\Theta/\Lambda)}$$

W.Ochs, J.Wosiek, PL B289 (1992) 159,B304 (1993) 144

Two-Particle Density

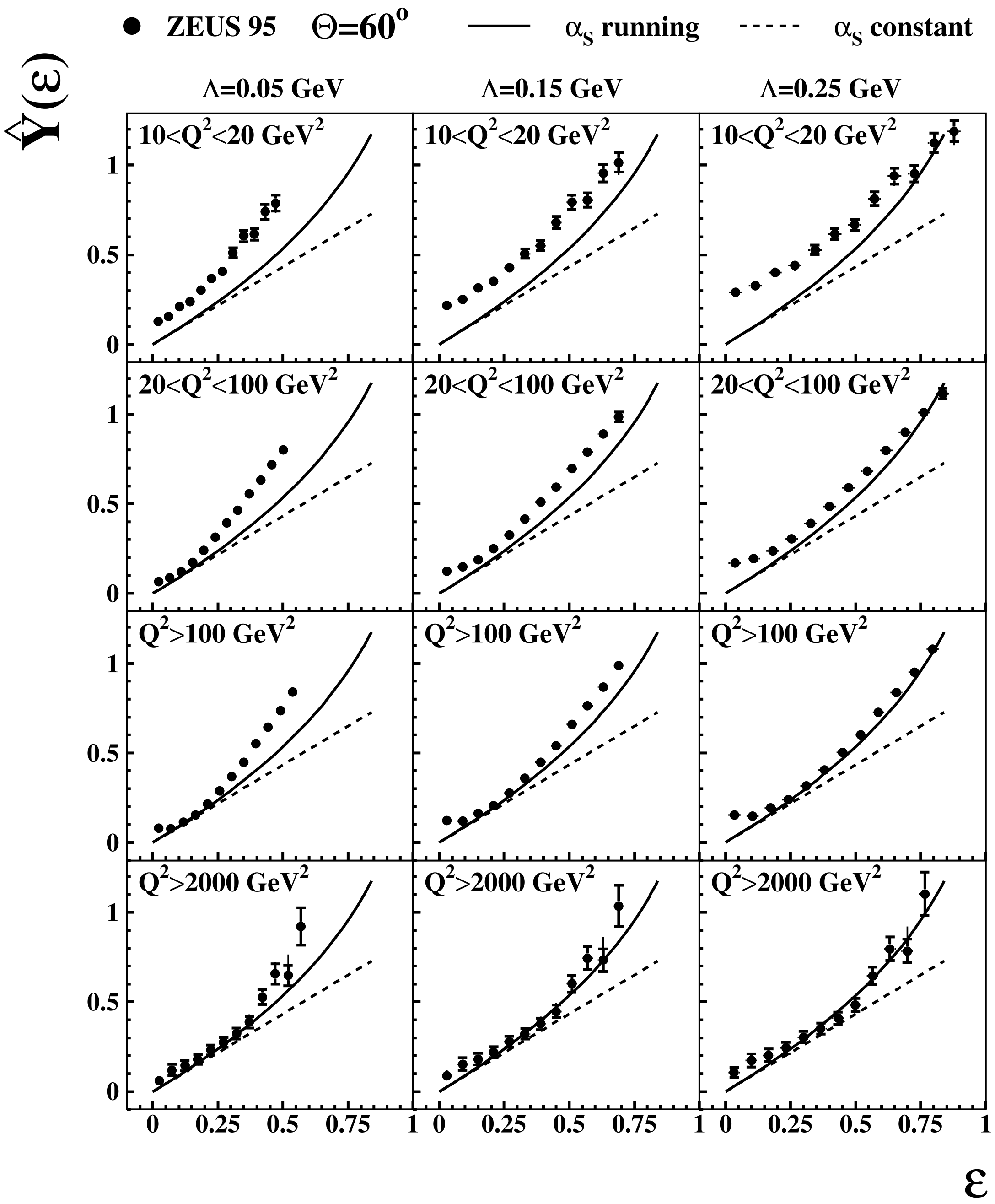
ZEUS Coll., EPJ C12 (2000) 53



Good description at high Q^2 and $\Lambda > 0.15$ GeV

e^+e^- and DIS data are in agreement

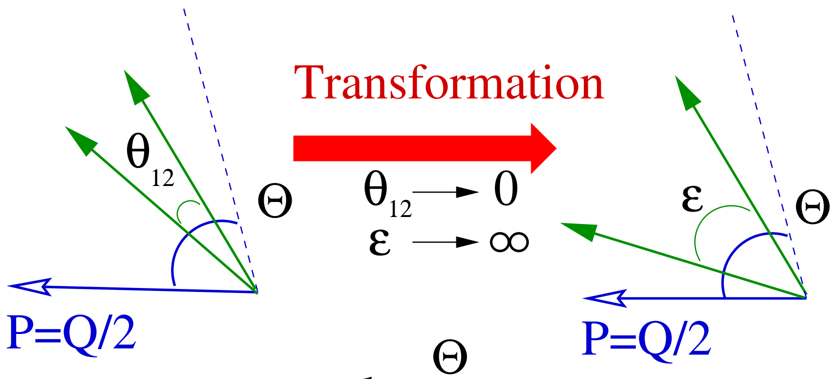
ZEUS Coll., EPJ C12 (2000) 53



Good description at high Q^2 and $\Lambda > 0.15$ GeV

Low Q^2 events: QCD cascade is not well developed ?

Two-Particle Correlations



$$\varepsilon = \frac{\ln \frac{\Theta}{\theta_{12}}}{\ln \frac{P \sin\Theta}{\Lambda}}$$

$$\rho(\varepsilon) = \frac{1}{N_{\text{ev}}} \frac{dn_{\text{pair}}}{d\varepsilon} \qquad Y \equiv \frac{\ln[\rho(\varepsilon)/\rho_{\text{mix}}(\varepsilon)]}{\sqrt{\ln(P \sin\Theta/\Lambda)}}$$

pQCD in DLLA + LPHD :

$$Y \simeq 2\beta(\omega(\varepsilon) - 2\sqrt{1-\varepsilon})$$

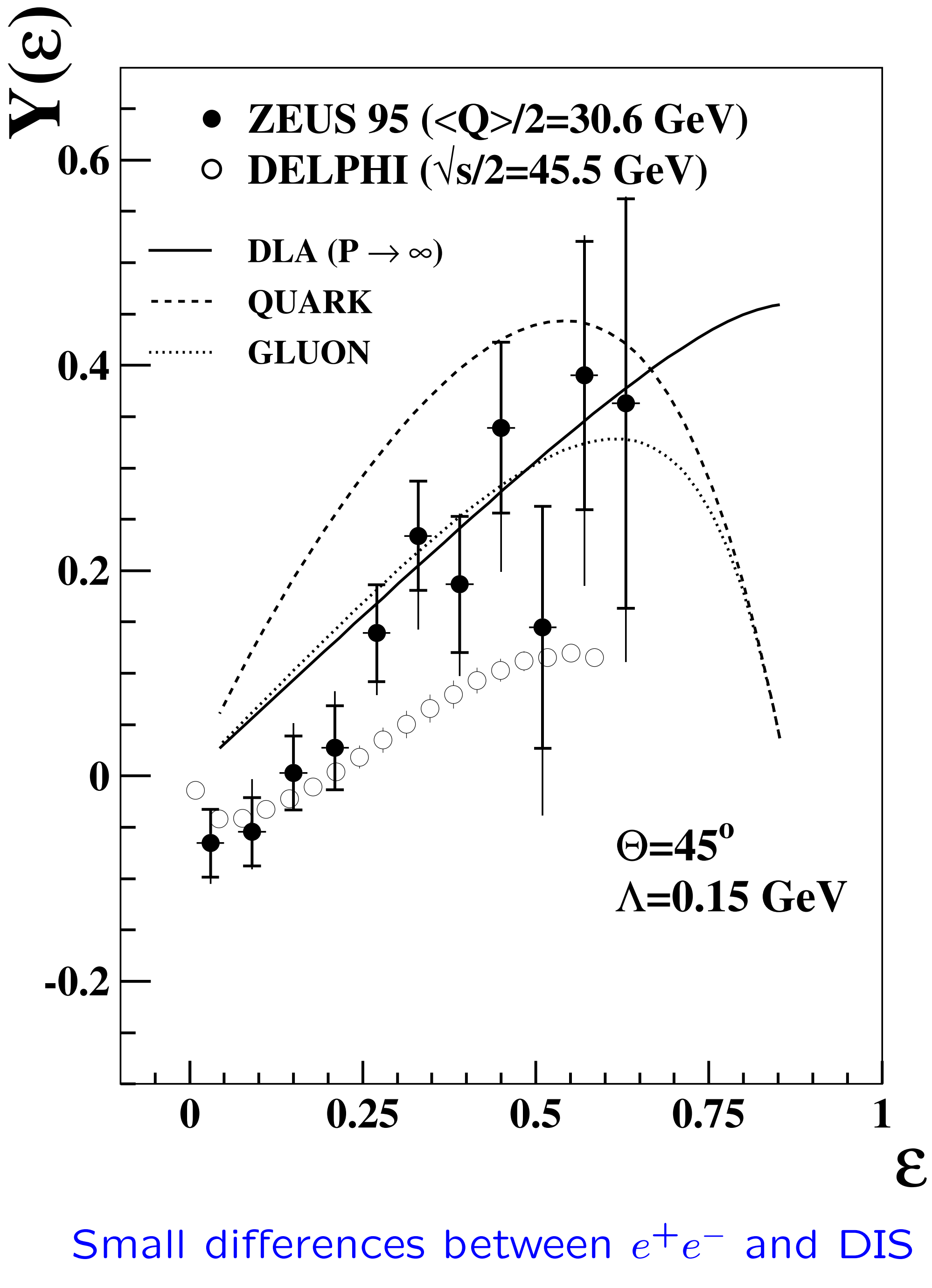
$$\omega(\varepsilon) = 2\sqrt{1-\varepsilon}\ (1 - \ln(1-\varepsilon)/8)$$

$$\beta = 6(11\,N_c - 2n_f)^{-1/2}, \qquad b = 2\beta\sqrt{\ln(P \sin\Theta/\Lambda)}$$

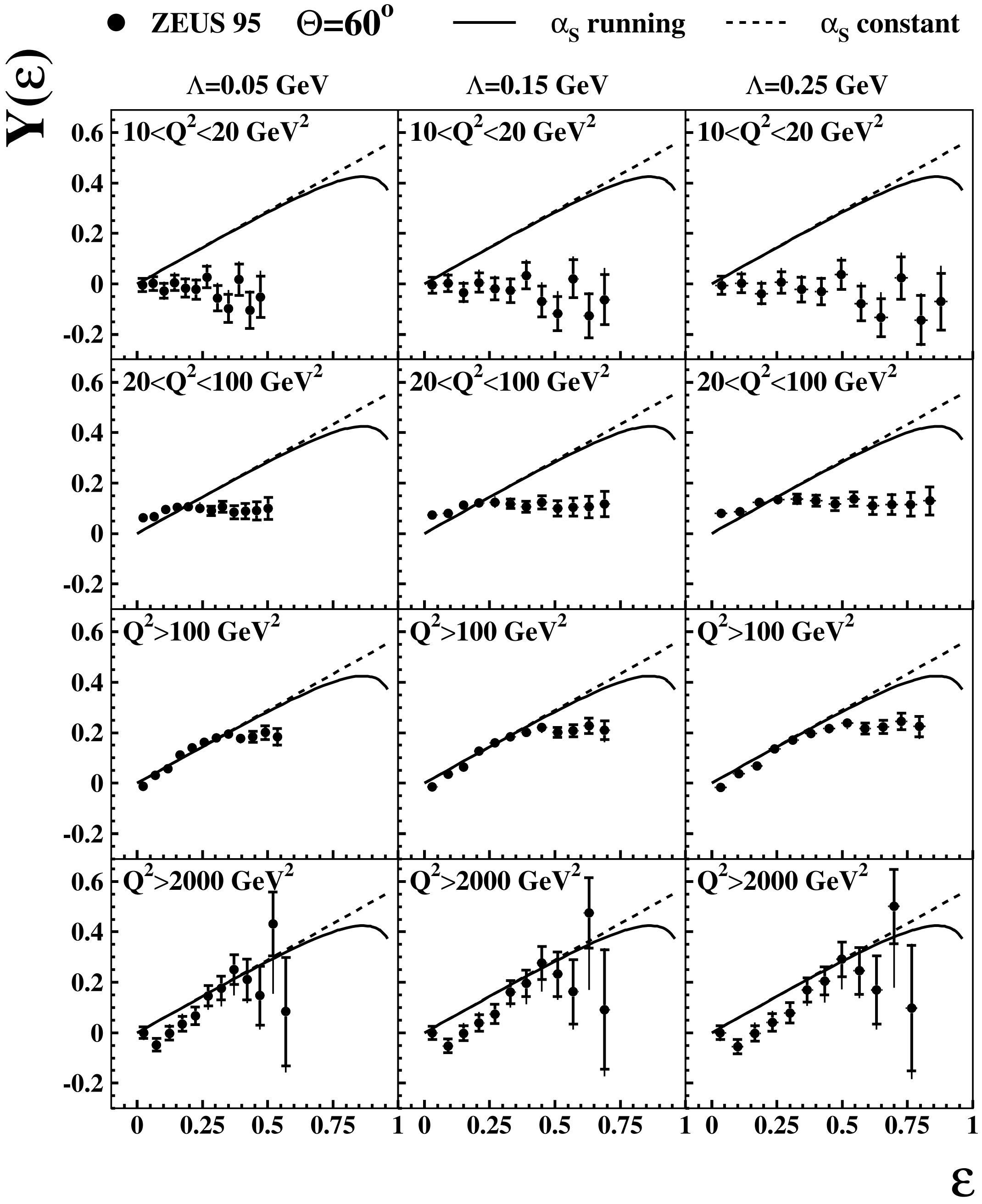
W.Ochs, J.Wosiek, PL B289 (1992) 159,B304 (1993) 144

Two-Particle Correlations in DIS

ZEUS Coll., EPJ C12 (2000) 53



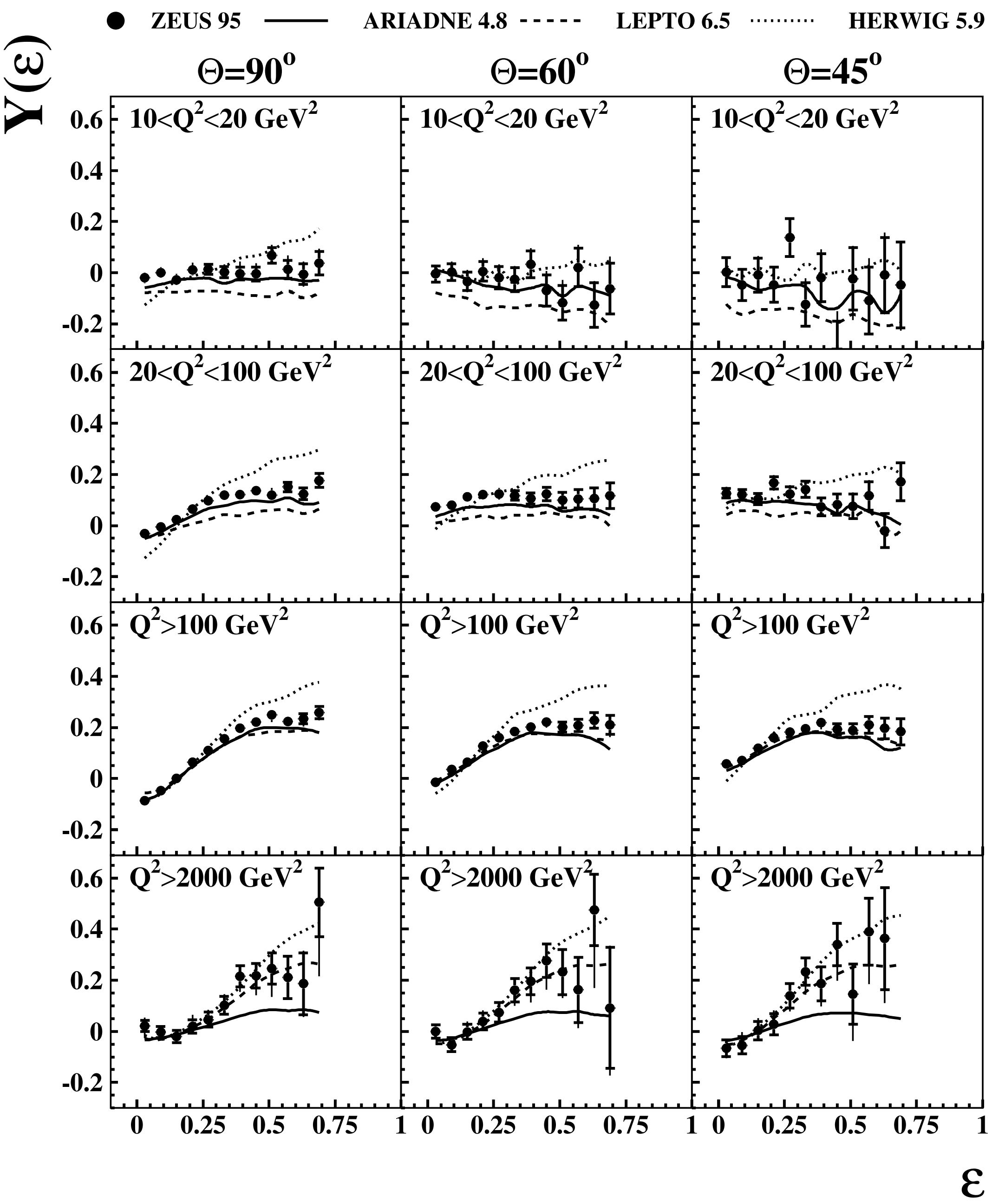
Small differences between e^+e^- and DIS



No correlations at $Q^2 < 20$ GeV^2

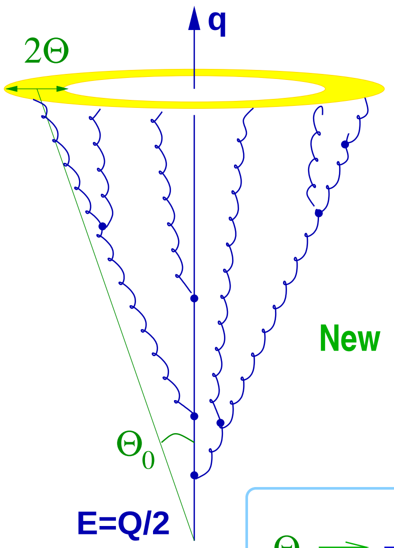
Significant discrepancies with DLLA at low Q^2

Two-Particle Correlations (data vs MC)



All MC models show discrepancies

Angular Moments



$P_n\ (2\Theta)$
probability to find
"n" gluons in the ring
of size 2Θ at Θ_0

New variable: $z = \ln \frac{\Theta_0}{\Theta} / a$

$a = \ln \frac{E\Theta_0}{\Lambda}$ = const

$\Theta \longrightarrow \frac{\Lambda}{E}$ $\qquad z \longrightarrow 1$

$\Theta \longrightarrow \Theta_0$ $\qquad z \longrightarrow 0$

Factorial moments:

$$F_q(z) = \langle n \rangle^{-q} \sum_{n=q}^{\infty} \frac{n!}{(n-q)!} P_n(z)$$

Experimental definition:

$$F_q(z) = \langle n \rangle^{-q} \langle n(n-1)\ldots(n-q+1) \rangle$$

DLLA QCD :

$$\ln \frac{F_q(z)}{F_q(0)} = z\, a\, (1 - D_q)(q-1)$$

Fluctuations are determined by D_q
($D_q = 1$, $F_q(z) = F_q(0)$ - no phase-space correlations)

QCD calculations for D_q:

- fixed-coupling regime

$$D_q = \gamma_0(E\Theta_0)\frac{q+1}{q}, \qquad \gamma_0(E\Theta_0) = \sqrt{2\,C_A\alpha_s(Q)/\pi}$$

- running coupling $\alpha_s(Q)$ calculations:

Yu. Dokshitzer and I.M. Dremin, Nucl. Phys. B402 (1993) 139

$$(a): \quad D_q \simeq \gamma_0(Q)\frac{q+1}{q}\left(1+\frac{q^2+1}{4q^2}z\right)$$

Ph. Brax, J.-L. Meunier and R. Peschanski, Z. Phys. C62 (1994) 649:

$$(b): \quad D_q \simeq 2\,\gamma_0(Q)\frac{q+1}{q}\left(\frac{1-\sqrt{1-z}}{z}\right)$$

W. Ochs and J. Wosiek, Phys. Lett. B289 (1992) 159, Z.Phys. C68 (1995) 269:

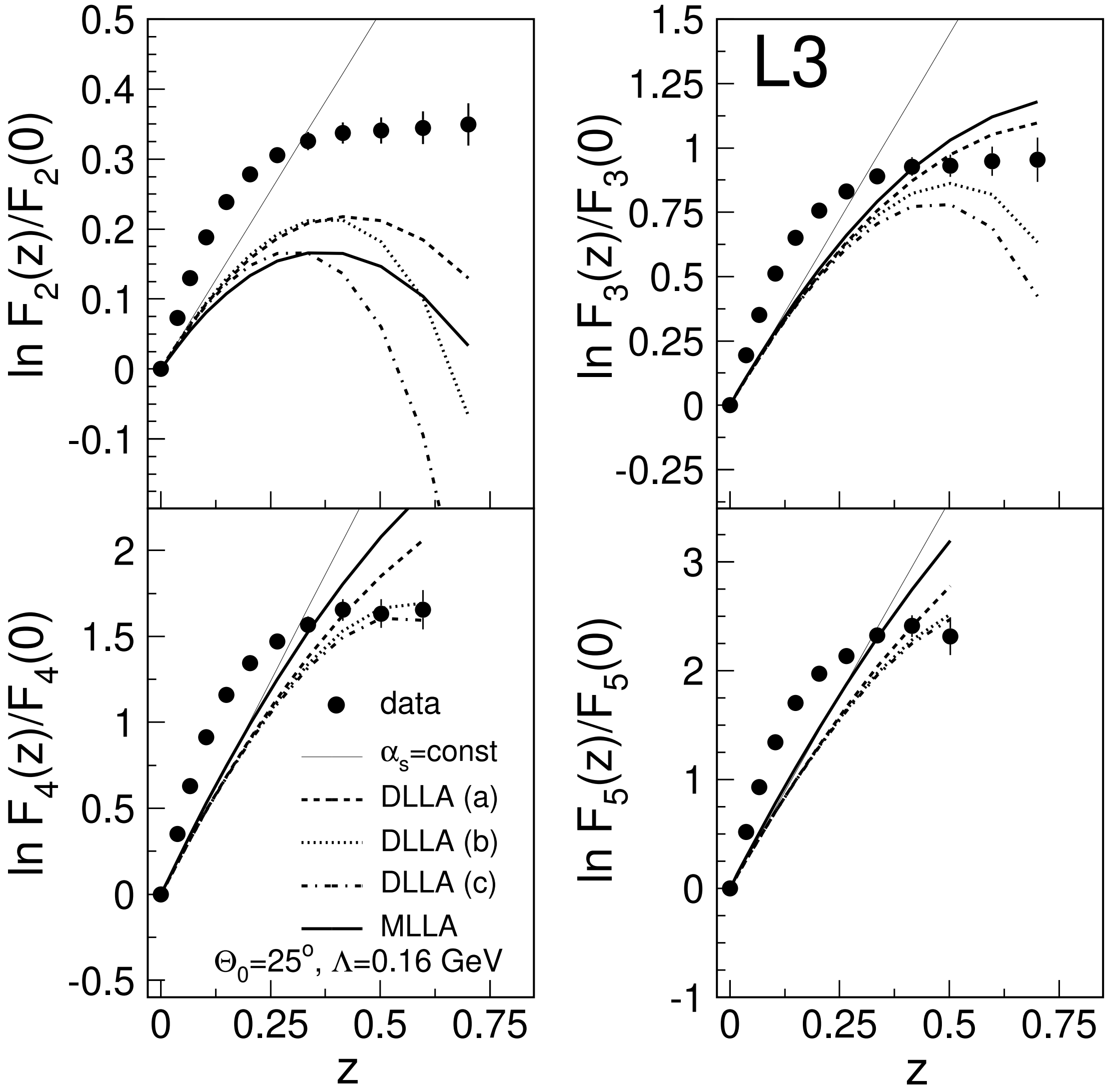
$$(c): \quad D_q \simeq 2\,\gamma_0(Q)\frac{q-w(q,z)}{z(q-1)}$$

$$w(q,z) = q\sqrt{1-z}\left(1-\frac{\ln(1-z)}{2q^2}\right).$$

MLLA corrections given in Nucl. Phys. B402 (1993) 139

Angular Moments in e^+e^-

L3 Coll., Phys. Lett. B428 (1998) 186



Significant differences with DLLA

Angular Moments in e^+e^-

DELPHI Coll., Phys. Lett. B457 (1999) 368

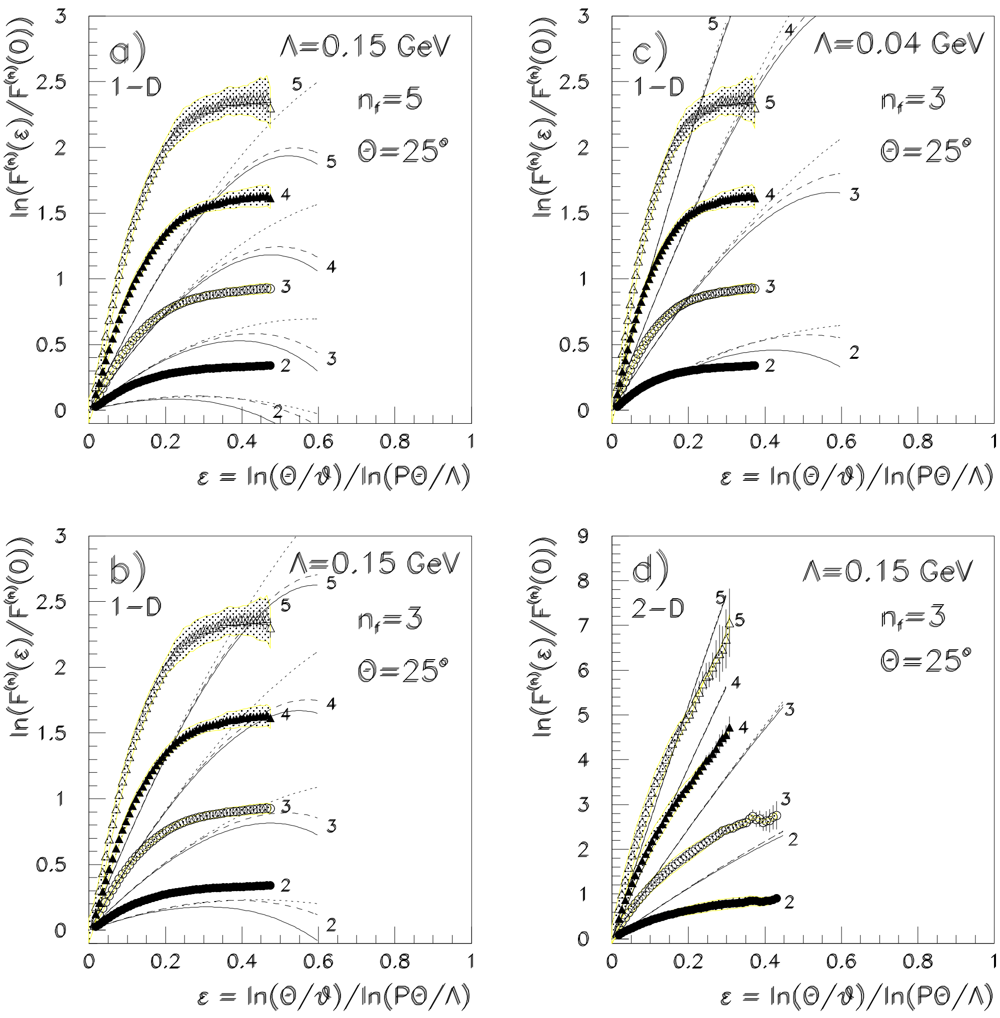


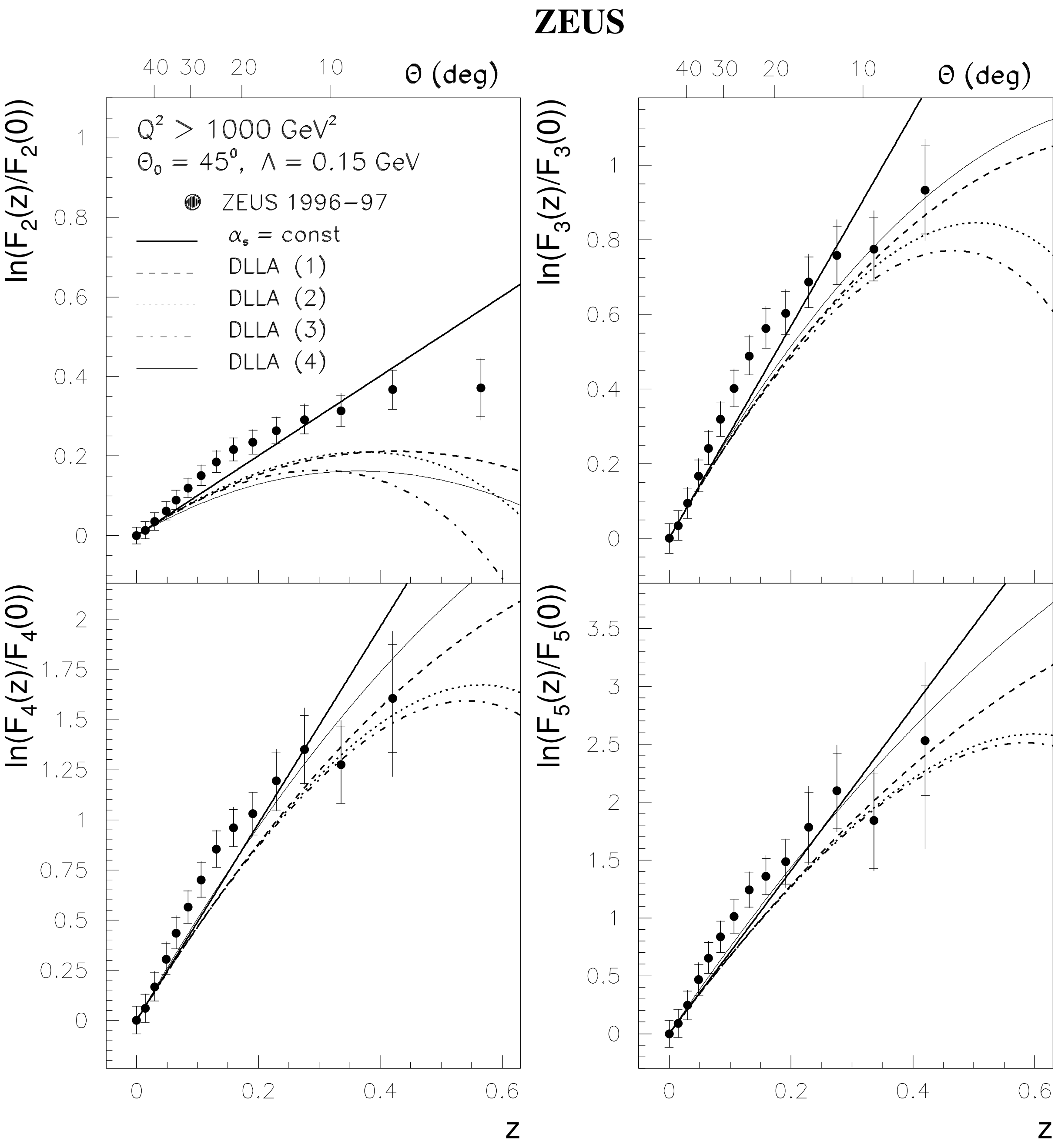
Figure 3: Factorial moments in 1-dimensional rings are compared with the analytical calculations of refs [4–6], eqs. 8 (solid lines), 11 (dashed lines), and 12 (dotted lines). The dependences on n_f and Λ are shown in a), b) and c). As a consistency test, 1- and 2-dimensional factorial moments are compared in b) and d) with same QCD parameters: note the different vertical scales. The orders 2 to 5 are indicated in all figures, the data are also distinguished by different symbols. The statistical errors are shown by the error bars, the shaded regions indicate the systematic errors. The 1-dimensional factorial moments agree very well with those measured by L3 [13].

Factorial moments rise faster in 2D

Significant differences with DLLA calculations

Angular Moments in DIS

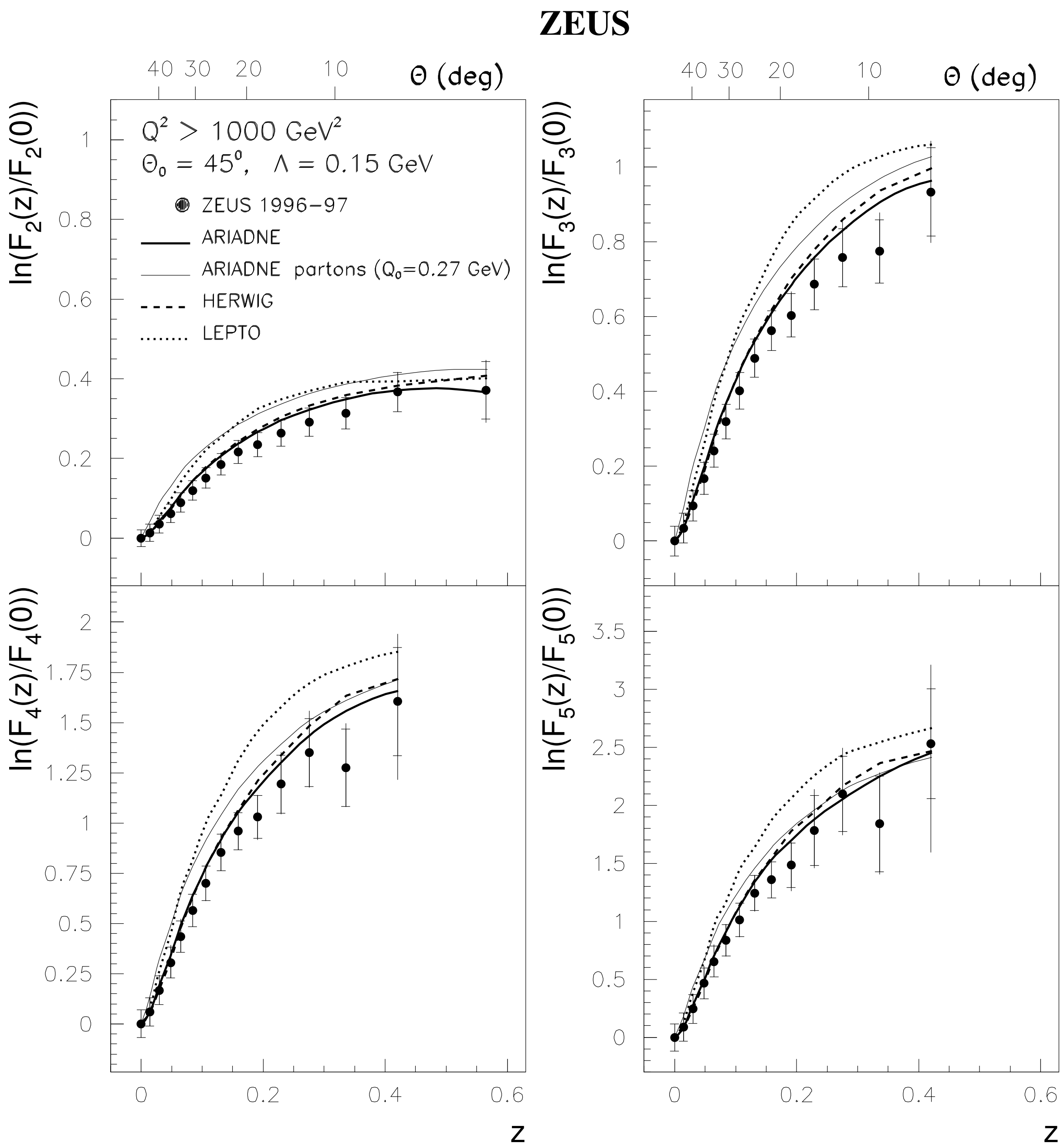
ZEUS Coll., Phys.Lett. B510 (2001) 36



In contrast to e^+e^-, higher-orders are better described by DLLA

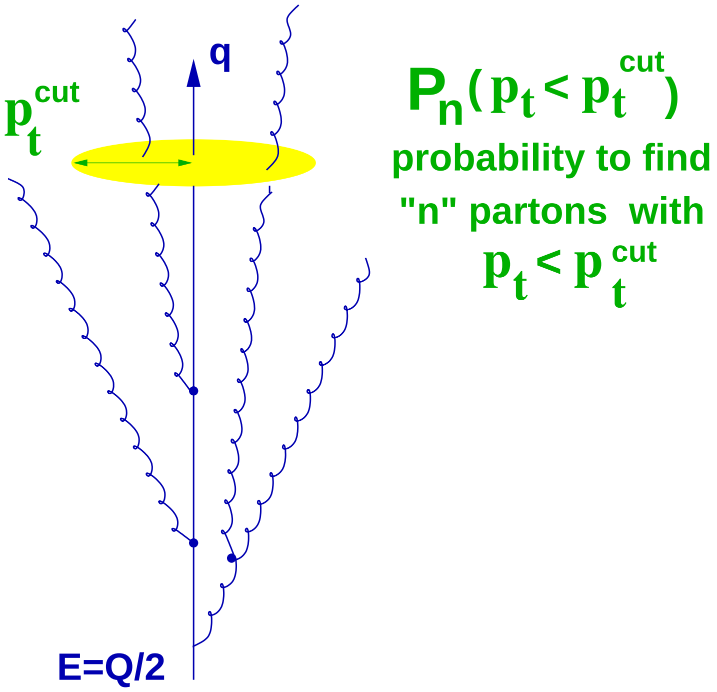
Angular Moments vs MC

ZEUS Coll., Phys.Lett. B510 (2001) 36



.. and MC models show some discrepancies

p_t^{cut} Moments



Factorial moments:

$$F_q(p_t < p_t^{cut}) = \langle n \rangle^{-q} \sum_{n=q}^{\infty} \frac{n!}{(n-q)!} P_n(p_t^{cut})$$

Experimental definition:

$$F_q(p_t < p_t^{cut}) = \langle n \rangle^{-q} \langle n(n-1)\ldots(n-q+1) \rangle$$

n number of gluons with $p_t < p_t^{cut}$

$$\boxed{\text{DLLA QCD calculations :}}$$

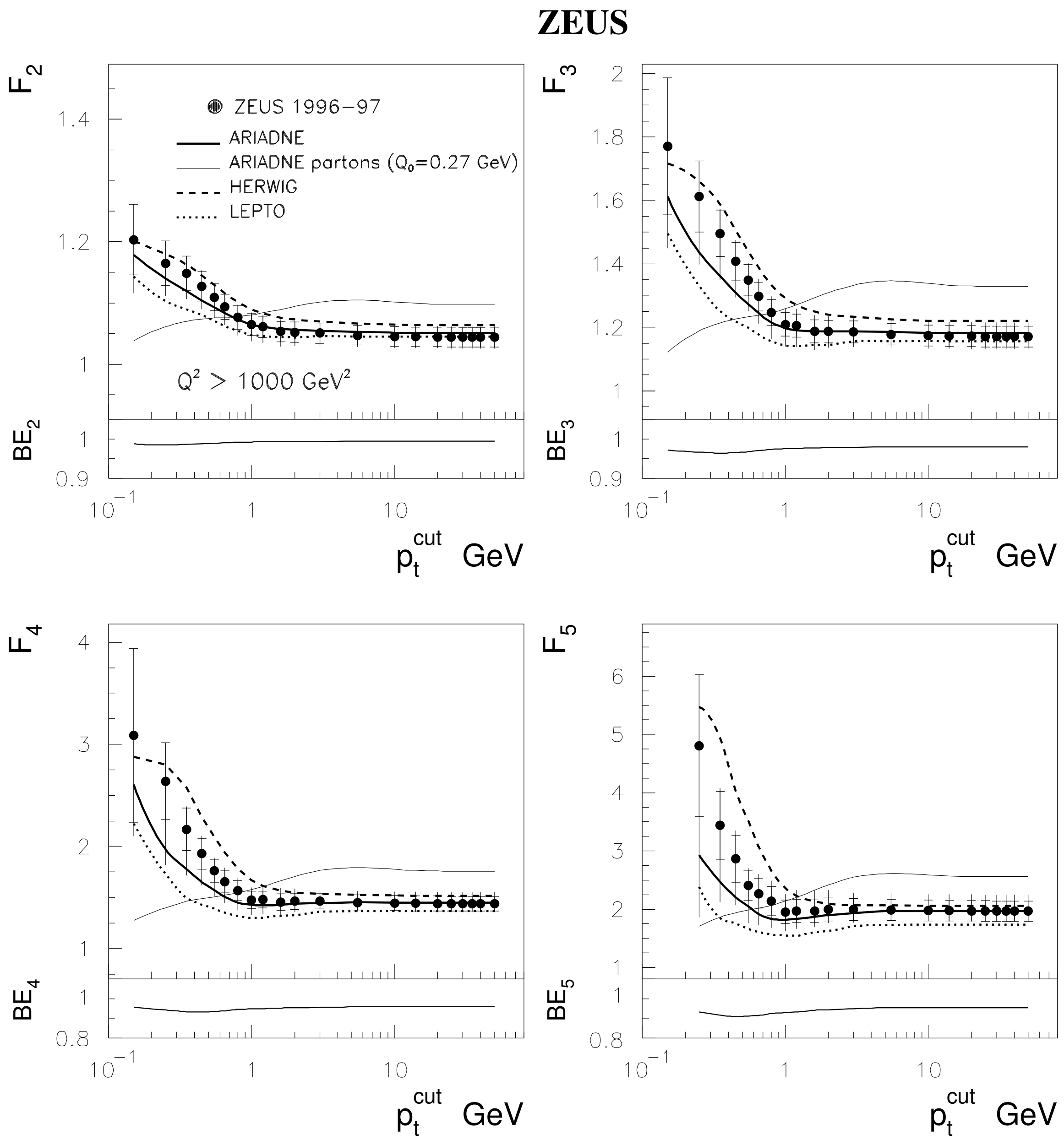
$$F_q(p_t^{cut}) \simeq 1 + \frac{q(q-1)}{6} \frac{\ln(p_t^{cut}/Q_0)}{\ln(E/Q_0)}$$

$F_q(p_t^{cut} \to Q_0) \to 1$ - Poisson statistics

S.Lupia, W.Ochs, J.Wosiek, Nucl. Phys. B540 (1999) 405

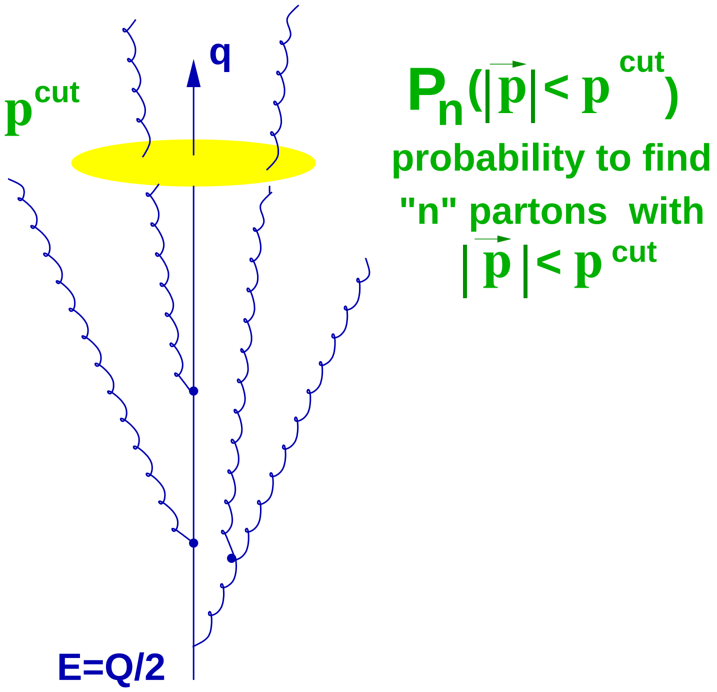
p_t^{cut} Moments

ZEUS Coll., Phys.Lett. B510 (2001) 36



Perturbative QCD, in conjunction with the LPHD hypothesis, fails on a qualitative level to describe the hadronic multiplicities

p^{cut} Moments



Factorial moments:

$$F_q(p < p^{cut}) = \langle n \rangle^{-q} \sum_{n=q}^{\infty} \frac{n!}{(n-q)!} P_n(p^{cut})$$

Experimental definition:

$$F_q(p < p^{cut}) = \langle n \rangle^{-q} \langle n(n-1)\ldots(n-q+1) \rangle$$

n number of gluons with $p < p^{cut}$

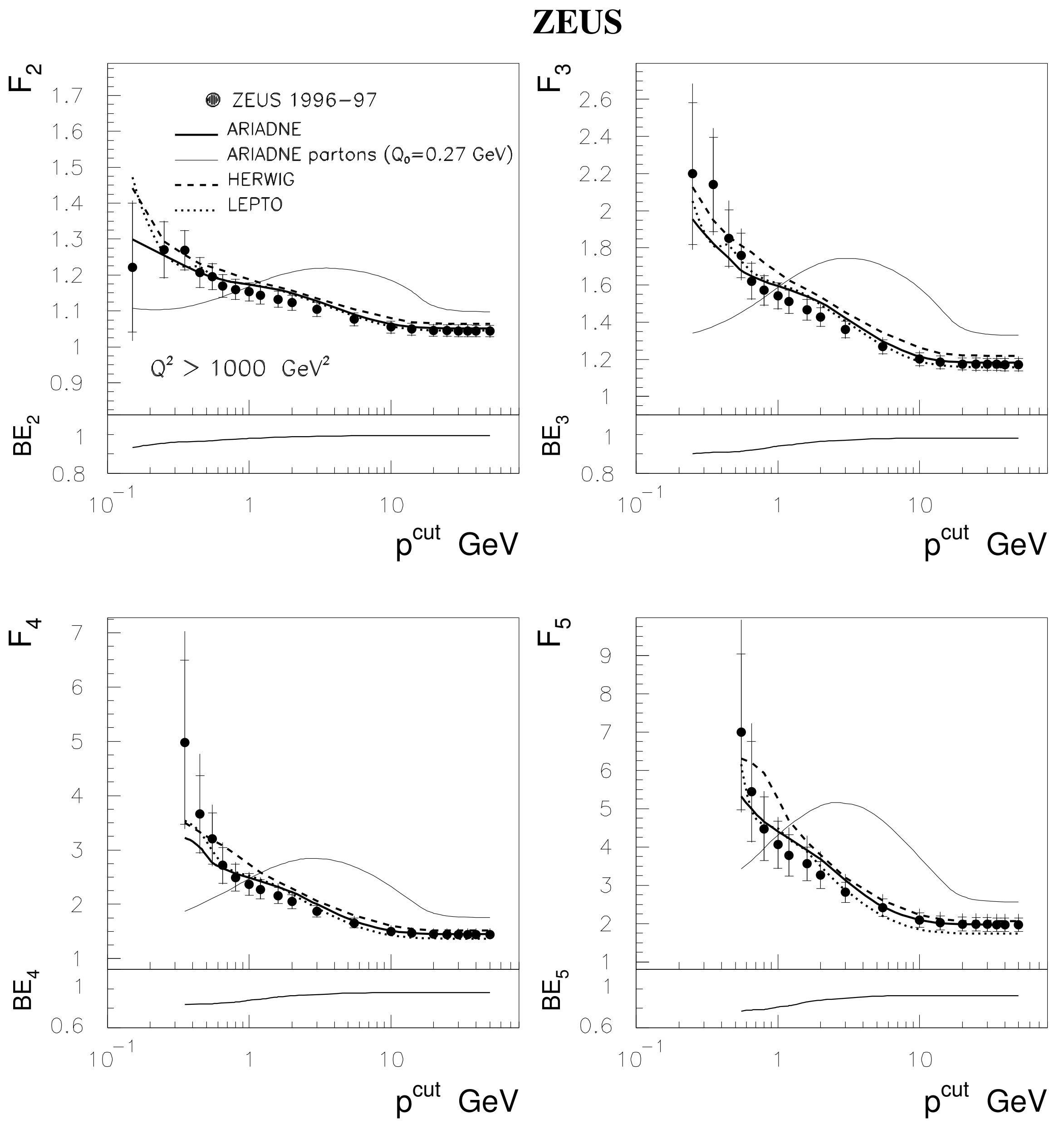
DLLA QCD :

$$F_q(p^{cut}) \simeq const$$

S.Lupia, W.Ochs, J.Wosiek, Nucl. Phys. B540 (1999) 405

p^{cut} Moments

ZEUS Coll., Phys.Lett. B510 (2001) 36



again large hadronisation effect !

Conclusions

- **DLLA QCD + LPHD:**
  Reasonably agrees with 2-particle density at high Q^2;
  Shows small discrepancies with 2-particle correlations;
  o MLLA is needed

- **Small differences between e^+e^- and DIS :**
  o need more statistics for DIS studies to confirm this !

- **Multiplicity moments disagree with DLLA:**
  o for angular moments - quantitative differences;
  o p_t and p moments show opposite trends !

- **Is this related to approximations in DLLA ?**

  o possibly for angular moments (see MC parton level)
  o *unlikely* for p_t and p-cut moments
  $\rightarrow$ MC parton-level ($\sim$ MLLA) shows the same discrepancies

  Thus:
- **Hadronisation has significant impact on correlations**
  o LPHD does not work for many-particle densities (and/or)
  o Many-particle inclusive densities are not sufficiently inclusive ?